Transmission Loss Allocation: A Comparison of Different Practical Algorithms

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***Abstract*—A pool-operated electricity market based on hourly auctions usually neglects network constraints and network losses while applying its market-clearing mechanism. This mechanism determines the accepted and nonaccepted energy bids as well as the hourly market-clearing prices. As a result, *ex post* procedures are needed to resolve network congestions and to allocate transmission losses to generators and demands. This paper focuses on transmission loss allocation procedures and provides a detailed comparison of four alternative algorithms: 1) *pro rata (PR)*; 2) marginal allocation; 3) unsubsidized marginal allocation; and 4) proportional sharing. A case study based on the IEEE RTS is provided. Different load scenarios covering a whole year are analyzed. Finally, conclusions and recommendations are stated.**

***Index Terms*—Electricity market, loss allocation, transmission losses.**

I. Introduction

AN APPROPRIATE method to clear the market in pool-based electricity markets is the use of hourly auctions [1]. Generators submit hourly energy bids and their corresponding prices to the power exchange (PX), while consumers submit hourly energy demands and their respective maximum buying prices. The PX market operator, on an hourly basis, builds the generator increasing stepwise curve of bids and the consumer decreasing stepwise curve of demands. The crossing of these two curves determines the hourly market-clearing price and allows determining how much energy each generator is allocated to produce. Hourly auctions are usually performed one day ahead. That is, the 24 auctions for tomorrow are usually performed today, so that enough time is available to check the technical feasibility of the results. This is, for instance, the case of the electricity market of mainland Spain [2].

The above-mentioned market-clearing procedure does not take into account the network and therefore losses are not explicitly accounted for. However, in real-time operation, consumer meters measure their actual consumptions, while generator meters measure their actual productions, i.e., the consumptions of customers plus the network losses. Naturally, the problem of "who should pay for losses" arises, and those payments constitute a substantial amount of money. In principle, both generators and consumers should pay for the losses because both do use the network and therefore are responsible for the losses incurred. Losses are, in fact, the result of the energy transactions through the transmission network in which generators and consumers are engaged.

Unfortunately, losses are nonlinear functions of line flows, and nonlinear electrical laws do not allow determining the amount of a line power flow which is the responsibility of a given generator or demand. Furthermore, if linearization techniques are used to allocate the flow of a given line to generators and demands, the cross terms associated with quadratic functions [$2xy$ versus x^2 and y^2 from $(x+y)^2$] do not allow assigning directly losses to generators and consumers [3].

These facts preclude the existence of a unique transmission loss allocation procedure. This paper focuses on the analysis of three families of procedures that have been reported in the technical literature: 1) *pro rata (PR)* procedures [4]; 2) marginal procedures [5]–[8]; and 3) proportional sharing procedures [9]–[15]. They are briefly described below. Other relevant approaches, such as circuit-based methods [16], and those devoted to bilateral transactions [17]–[20], are outside the scope of this paper.

1) PR Procedures: First, losses are globally assigned to generators and consumers, for instance 50% of losses are allocated to each category. Then, a proportional allocation rule is used: the losses allocated to a generator (consumer) are proportional to its corresponding level of energy generation (consumption). A PR procedure is currently used in the electricity market of mainland Spain where 100% of losses are allocated to consumers [2].

2) Marginal Procedures: Losses are assigned to generators and demands through the so-called incremental transmission loss (ITL) coefficients [5], [6]. A normalization is performed after the assignment because this allocation procedure typically results in over-recovery. Reference [7] provides analyses and results from a practical implementation of a marginal allocation procedure in the Norwegian electric system. An integral method has been recently presented in [8] where a distributed slack bus is used.

3) Proportional Sharing Procedures: The use of the results of a converged power flow plus a linear proportional sharing principle [9]–[15] make it possible for the allocation of losses to generators and consumers. This principle states that "the power flow reaching a bus from any power line splits among the lines evacuating power from the bus proportionally to their corresponding power flows," which is neither provable nor disprovable.

Pro rata procedures are simple to understand and implement. However, they "ignore" the network. That is, two identical de-

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mands located respectively near generating buses and far away from these buses are equally treated, and this is unfair for the load located near the generating buses.

The standard marginal procedure based on ITL coefficients depends on the selection of the slack bus because ITL coefficients do depend on the slack bus. The ITL coefficient of the slack bus is zero by definition, thus the slack bus is allocated no losses. This is a drastic limitation for this method that requires that pool agents agree beforehand on the selection of the slack bus. Furthermore, ITL coefficients can be either positive or negative which may result in the allocation of negative losses to certain buses. And this may be interpreted as cross subsidies. This marginal procedure can be modified to avoid subsidies. This modification will be referred to as *unsubsidized marginal allocation*.

Proportional sharing procedures, on top of electrical laws, require the assumption of the proportional sharing principle. Using this principle, losses are allocated by linear procedures. To allocate losses to demands, the method relies on a simple principle: losses associated with every line whose flow enters a given bus are transferred to the lines whose flows leave the bus (or demands in that bus) proportionally to the flows of those lines (the flows of which leave the bus). It should be noted that a systematic application of this principle originates that all losses are allocated to demands. Analogously, in order to allocate losses to generators, the method relies on a simple principle: losses associated with every line whose flow leaves a given bus are transferred to the lines whose flows enter the bus (or generations in that bus) proportionally to the flows of those lines (whose flows enter the bus). It should be noted that a systematic application of this principle originates that all losses are allocated to generators. The sole information required to apply this method is the real power flow and the losses in every line, and the power generated or consumed in every bus.

It should be emphasized that the purpose of a loss allocation procedure is to assign the cost of losses to generators and demands. It is therefore an *ex post* economic mechanism that does not interfere with the technical functioning of the transmission network.

Due to the fact that no unique or ideal procedure exists, any loss allocation algorithm should have most of the desirable properties stated below:

1) to be consistent with the results of a power flow;
2) to depend on the amount of energy either produced or consumed;
3) to depend on the relative location in the transmission network;
4) to avoid volatility;
5) to provide appropriate economic marginal signals;
6) to be easy to understand;
7) to be simple to implement.

The remaining of this paper is organized as follows. Section II presents in detail four transmission loss allocation procedures. Section III is a detailed case study based on the IEEE RTS [21] in which the four algorithms are compared. Section IV provides conclusions and recommendations.

II. TRANSMISSION LOSS ALLOCATION METHODS

First, note that the sum of all generations is equal to the sum of all demands plus the losses. That is

$$P_G = P_D + L, \quad P_G = \sum_{i=1}^{N_G} P_{Gi}, \quad P_D = \sum_{j=1}^{N_D} P_{Dj} \qquad (1)$$

where

P_G total active power generated;
P_{Gi} power output of generators of bus i;
P_D total active power demand;
P_{Dj} active power demanded by consumers of bus j;
L transmission power losses;
N_G number of generating buses;
N_D number of demand buses.

For simplicity and without loss of generality, it is assumed that in every bus there are at most one generator and one demand. Therefore, no distinction will be made henceforth between generator i, load i, and bus i.

The considered transmission loss allocation methods are described in the four subsections below.

A. Pro Rata Allocation (PR)

The PR method proportionally allocates 50% of losses to the demands and 50% to the generators, that is

$$L_{Gi} = \frac{L}{2}\frac{P_{Gi}}{P_G}, \quad L_{Dj} = \frac{L}{2}\frac{P_{Dj}}{P_D} \qquad (2)$$

where L_{Gi} are the losses allocated to the generator i, and L_{Dj} are the losses allocated to the demand j.

Generation and demand loss allocation factors are computed, respectively, as

$$L_{Gi} = \frac{L}{2}\frac{P_{Gi}}{P_G} = K_G P_{Gi}, \quad K_G = \frac{1}{2}\frac{L}{P_G} \qquad (3)$$

$$L_{Dj} = \frac{L}{2}\frac{P_{Dj}}{P_D} = K_D P_{Dj}, \quad K_D = \frac{1}{2}\frac{L}{P_D}. \qquad (4)$$

It should be noted that generation loss allocation factors K_G are identical for all buses, and demand loss allocation factors K_D are also identical for all buses. Additionally, it should be noted that losses allocated to generators and demands are always positive.

B. Marginal Allocation (ITL)

This method uses ITL coefficients to proportionally allocate losses to generators and demands. ITLs are easily obtained from a converged power flow [5], [6]. The ITL of a given bus provides the change in total losses produced by an incremental change in the power injected in that bus. Therefore

$$K_i = \frac{\partial L}{\partial (P_{Gi} - P_{Di})} \qquad (5)$$

where K_i is the ITL corresponding to bus i. It should be noted that the ITL of the slack bus is zero by definition.

First computations of the losses allocated to generator i and demand j are, respectively,

$$L_{Gi} = P_{Gi}\frac{\partial L}{\partial P_{Gi}} = P_{Gi}K_i \tag{6}$$

$$L_{Dj} = P_{Dj}\frac{\partial L}{\partial P_{Dj}} = -P_{Dj}K_j. \tag{7}$$

However, and as a result of nonlinearities, the sum of these allocated losses (L') does not match total actual (measured) losses L, that is

$$\begin{aligned} L &\neq \sum_{i=1}^{N_G} L_{Gi} + \sum_{j=1}^{N_D} L_{Dj} \\ &= \sum_{i=1}^{N_G} P_{Gi}K_i - \sum_{j=1}^{N_D} P_{Dj}K_j = L'. \end{aligned} \tag{8}$$

Therefore, a normalization procedure is used to allocate the exact amount of losses L

$$\begin{aligned} L &= L'\frac{L}{L'} = \left(\sum_{i=1}^{N_G} P_{Gi}K_i - \sum_{j=1}^{N_D} P_{Dj}K_j\right)\frac{L}{L'} \\ &= \sum_{i=1}^{N_G} P_{Gi}K'_i - \sum_{j=1}^{N_D} P_{Dj}K'_j \end{aligned} \tag{9}$$

where $K'_i = K_i(L/L')$ is the normalized ITL coefficient for bus i.

Finally, losses allocated to every generator and demand are, respectively,

$$L'_{Gi} = P_{Gi}K'_i, \quad L'_{Dj} = -P_{Dj}K'_j. \tag{10}$$

It should be noted that this marginal procedure may allocate negative losses to either generators or demands, and these negative losses can be interpreted as cross subsidies.

C. Unsubsidized Marginal Allocation

The unsubsidized ITL (U-ITL) method modifies in a consistent manner ITL coefficients so that negative losses are avoided. As a result, a set of ITLs is defined for generators and a different one for demands. It should be emphasized that the purpose of this method is to allocate the cost of losses, not to explain physical facts.

ITL coefficients, computed for a given slack bus, can easily be referred to a different slack bus by defining a translation coefficient β $(0 \leq \beta \leq 1)$ [4]. This is used below.

Total losses can be computed as

$$L = \sum_{i=1}^{N} K'_i P_i \tag{11}$$

where

N	number of buses;
K'_i	normalized ITL coefficient of bus i;
P_i	injected active power in bus i $(P_i = P_{Gi} - P_{Di})$.

Total losses can also be expressed as

$$L = \sum_{i=1}^{N}(P_{Gi} - P_{Di}) = \sum_{i=1}^{N} P_i. \tag{12}$$

Multiplying (11) by β $(0 \leq \beta \leq 1)$ and (12) by $1-\beta$, and adding both, total losses can be expressed as

$$L = \sum_{i=1}^{N} \beta K'_i P_i + \sum_{i=1}^{N}(1-\beta)P_i \tag{13}$$

which results in

$$L = \sum_{i=1}^{N} \left[\beta K'_i + (1-\beta)\right] P_i = \sum_{i=1}^{N} K_i P_i \tag{14}$$

where $\beta K'_i + (1-\beta)$ constitutes a new ITL coefficient $K_i = \beta K'_i + (1-\beta)$.

In respect to the generation, a change of slack bus is performed in such a way that the generator ITL coefficient with smallest value becomes zero. This makes it impossible to assign negative losses to generators. This is accomplished as stated below.

Let K'_{Gk} be the normalized generation ITL coefficient with the smallest value, the translation coefficient β_G is then computed as

$$K_{Gk} = 0 = \beta_G K'_{Gk} + (1-\beta_G) \tag{15}$$

and

$$\beta_G = \frac{1}{1-K'_{Gk}}.$$

New ITL coefficients for generators are therefore

$$K_{Gi} = \beta_G K'_{Gi} + (1-\beta_G). \tag{16}$$

Those coefficients are again normalized to allocate 50% of losses to generators.

In respect to demands, the translation coefficient β_D is computed from

$$K_{Dm} = 0 = \beta_D K'_{Dm} + (1-\beta_D) \tag{17}$$

where K'_{Dm} is the demand ITL coefficient with the highest value. Equation (17) guarantees that no demand gets allocated negative losses. Therefore, demand ITL coefficients become all negative.

From (17), $\beta_D = 1/(1-K'_{Dm})$.

Furthermore, new demand ITL coefficients become

$$K_{Dj} = \beta_D K'_{Dj} + (1-\beta_D). \tag{18}$$

Finally, those coefficients are again normalized to allocate 50% of losses to demands.

D. Proportional Sharing Allocation

For the reader's convenience, this subsection briefly summarizes Bialek's proportional sharing algorithm (PS) [9], [10].

Losses are first allocated to demands and then to generators. In respect to demands, a total gross demand including losses P_D^G is defined as

$$P_D^G = P_D + L \quad \text{and} \quad P_D^G = \sum_{j=1}^{N_D} P_{Dj}^G \tag{19}$$

where P_{Dj}^G is the gross demand of bus j.

The total gross demand must equal the total generation so that $P_G = P_D^G$. Using the proportional sharing principle, the power balance in every bus of an equivalent lossless network becomes

$$P_i^G = P_{Gi} + \sum_{j \in \alpha_i} c_{ji} P_j^G, \qquad \forall i = 1, \ldots, N \tag{20}$$

with

$$c_{ji} = \frac{P_{ji}^G}{P_j^G} \approx \frac{P_{ji}}{P_j} \tag{21}$$

where

P_i^G	gross power injected in bus i;
P_{Gi}	generation in bus i;
$\sum_{j \in \alpha_i} c_{ji} P_j^G$	power flow reaching bus i from lines connected to it;
α_i	set of buses from which power flows toward bus i;
P_{ji}^G	gross power flow from j to i;
P_{ji}	actual power flow from j to i (measured in j);
P_j	actual power injection in bus j.

Equation (20) constitutes a system of linear equations that is solved easily for P_i^G, $i = 1, \ldots, N$. Gross demands and losses are then computed, respectively, as

$$P_{Dj}^G = \frac{P_j^G}{P_j} P_{Dj} \quad \text{and} \quad L_{Dj} = P_{Dj}^G - P_{Dj}. \tag{22}$$

Analogously, losses are assigned to generators. Total gross generation including losses P_G^G is defined as

$$P_G^G = P_G + L \quad \text{and} \quad P_G^G = \sum_{i=1}^{N_G} P_{Gi}^G \tag{23}$$

where P_{Gi}^G is the gross generation of bus i (including losses).

This gross generation must equal total demand, so that $P_G^G = P_D$. Using the proportional sharing principle, the power balance in bus i, of an equivalent lossless network becomes

$$P_i^G = P_{Di} + \sum_{j \in \gamma_i} c_{ji} P_j^G, \qquad \forall i = 1, \ldots, N \tag{24}$$

where

P_i^G	gross power injected in bus i;
P_{Di}	demand in bus i;
$\sum_{j \in \gamma_i} c_{ji} P_j^G$	power flow leaving bus i;
γ_i	set of buses drawing power from bus i.

TABLE I
DATA OF SCENARIOS

Scenarios	Hour	Week	Day	# Hours
Peak, Weekday, Winter	8:00-9:00	50	5	595
Shoulder, Weekday, Winter	21:00-22:00	46	4	935
Valley, Weekday, Winter	3:00-4:00	3	2	510
Peak, Weekday, Spring	19:00-20:00	12	2	225
Shoulder, Weekday, Spring	14:00-15:00	11	5	630
Valley, Weekday, Spring	1:00-2:00	9	1	225
Peak, Weekday, Summer	10:00-11:00	24	5	455
Shoulder, Weekday, Summer	19:00-20:00	27	3	780
Valley, Weekday, Summer	4:00-5:00	19	4	325
Peak, Weekday, Fall	10:00-11:00	32	1	325
Shoulder, Weekday, Fall	7:00-8:00	34	3	910
Valley, Weekday, Fall	1:00-2:00	41	1	325
Peak, Weekend, Winter	21:00-22:00	51	7	102
Shoulder, Weekend, Winter	23:00-24:00	47	6	510
Valley, Weekend, Winter	7:00-8:00	7	6	204
Peak, Weekend, Spring	19:00-20:00	12	6	54
Shoulder, Weekend, Spring	8:00-9:00	17	6	288
Valley, Weekend, Spring	2:00-3:00	15	7	90
Peak, Weekend, Summer	17:00-18:00	23	6	52
Shoulder, Weekend, Summer	11:00-12:00	29	7	442
Valley, Weekend, Summer	3:00-4:00	18	7	130
Peak, Weekend, Fall	11:00-12:00	33	7	78
Shoulder, Weekend, Fall	9:00-10:00	35	7	416
Valley, Weekend, Fall	2:00-3:00	36	6	130

Equation (24) constitutes a system of linear equations that can be solved easily for P_i^G, $i = 1, \ldots, N$. New generations and losses are then computed, respectively, as

$$P_{Gi}^G = \frac{P_i^G}{P_i} P_{Gi} \quad \text{and} \quad L_{Gi} = P_{Gi} - P_{Gi}^G. \tag{25}$$

In order to assign 50% of losses to the generation and 50% to the demand, the final generation and demand per bus are computed as

$$P'_{Gi} = \frac{P_{Gi}^G + P_{Gi}}{2} \quad \text{and} \quad P'_{Dj} = \frac{P_{Dj}^G + P_{Dj}}{2}. \tag{26}$$

Final losses assigned to every generator and demand are, respectively,

$$L'_{Gi} = P_{Gi} - P'_{Gi} \quad \text{and} \quad L'_{Dj} = P'_{Dj} - P_{Dj}. \tag{27}$$

Finally, generation and demand loss allocation factors are, respectively, computed as

$$K_{Gi} = 1 - \frac{P'_{Gi}}{P_{Gi}} \quad \text{and} \quad K_{Dj} = \frac{P'_{Dj}}{P_{Dj}} - 1. \tag{28}$$

III. CASE STUDY

The well-known IEEE RTS [21] is used to compare the four transmission loss allocation procedures considered in this paper. The IEEE RTS comprises 24 buses and 33 lines. The number of scenarios considered is 24, corresponding to peak, shoulder, and valley demands, of a weekday and a weekend day, for the four seasons. Data of the scenarios is shown in Table I. Minor modifications are introduced in the reactive power limits of the generators. For every scenario, a power flow is solved using the PowerWorld tool [22]. Power flow data and results provide the input data for the four loss allocation algorithms. Results for the

TABLE II
Percentage of Losses Allocated to Every Load Using the Four Compared Loss Allocation Procedures

Percentage of yearly losses [%]				
	Procedures			
Load bus #	PR	ITL	U-ITL	PS
1	1.89	-0.77	2.19	0.30
2	1.70	-0.61	1.99	0.03
3	3.16	-3.03	3.11	4.97
4	1.30	1.73	2.19	1.71
5	1.25	1.44	2.03	2.02
6	2.39	6.89	5.15	8.55
7	2.19	-0.66	2.60	0.00
8	3.00	5.67	5.56	4.99
9	3.07	1.63	4.42	5.15
10	3.42	3.68	5.49	5.66
13	4.65	0.00	5.95	3.34
14	3.40	-2.07	3.72	5.25
15	5.56	-17.12	1.90	3.28
16	1.75	-4.94	0.74	1.21
18	5.84	-24.56	0.00	0.34
19	3.18	-7.49	1.78	2.84
20	2.25	-5.56	1.18	0.36
TOTAL	50.00	-45.77	50.00	50.00

TABLE III
Losses Allocated to Every Load Using the Four Compared Loss Allocation Procedures

Total yearly losses [MWh]				
	Procedures			
Load bus #	PR	ITL	U-ITL	PS
1	3900	-1590	4501	627
2	3503	-1259	4094	52
3	6500	-6246	6409	10234
4	2672	3570	4502	3513
5	2564	2973	4182	4156
6	4911	14178	10594	17598
7	4514	-1367	5353	0
8	6175	11664	11441	10269
9	6319	3356	9100	10605
10	7041	7572	11307	11642
13	9569	0	12235	6870
14	7005	-4266	7659	10811
15	11447	-35233	3916	6745
16	3611	-10166	1524	2491
18	12023	-50536	0	709
19	6536	-15425	3664	5842
20	4622	-11435	2431	748
TOTAL	102912	-94210	102912	102912

24 scenarios are aggregated weighted with their corresponding time spans (see Table I) to provide results for the whole year.

Table II provides, for every load, the percentage of total yearly losses allocated using the four compared procedures. Table III provides the same information as Table II for actual losses in megawatthours. Table IV provides for every generating bus the percentage of total yearly losses allocated using the four compared procedures. Table V provides the same information as Table IV for actual losses in megawatthours.

From the above results, it is worth noting the following:

1) The demand is heavily subsidized by the generation when using the ITL procedure.
2) The ITL procedure presents very high volatility (as it is apparent when comparing results from different scenarios).
3) Bus 13 does not get allocated losses when using the ITL procedure because it is the slack bus.
4) Demand bus 18 and generation bus 13 do not get allocated losses when applying the U-ITL method because they are the demand and generation buses, respectively, used for the slack bus translation.
5) The PR procedure generates allocation results significantly different than those produced by other algorithms.

TABLE IV
Percentage of Losses Allocated to Every Generating Bus Using the Four Compared Procedures

Percentage of yearly losses [%]				
	Procedures			
Generating bus #	PR	ITL	U-ITL	PS
1	2.98	1.23	0.42	1.19
2	2.98	1.08	0.37	3.64
7	4.15	1.28	0.43	1.76
13	3.02	0.00	0.00	1.04
15	3.72	11.61	3.98	2.14
16	2.68	7.66	2.63	2.30
18	6.93	29.49	10.12	3.85
21	6.93	30.84	10.58	7.88
22	5.19	30.89	10.60	10.22
23	11.42	31.69	10.87	15.98
TOTAL	50.00	145.77	50.00	50.00

TABLE V
Losses Allocated to Every Generating Bus Using the Four Compared Loss Allocation Procedures

Total yearly losses [MWh]				
	Procedures			
Generating bus #	PR	ITL	U-ITL	PS
1	6128	2531	864	2444
2	6127	2233	762	7489
7	8550	2625	894	3626
13	6225	0	0	2138
15	7659	23896	8197	4411
16	5522	15757	5405	4741
18	14250	60704	20824	7929
21	14250	63469	21773	16213
22	10688	63586	21813	21028
23	23513	65233	22380	32893
TOTAL	102912	300034	102912	102912

IV. CONCLUSIONS

From the different case studies analyzed, the following conclusions are drawn:

1) The ITL method presents high volatility and negative losses. Furthermore, it may present a high loss allocation imbalance between generation and demand, e.g., generators are allocated 146% of losses and demands −46%.
2) The U-ITL method retains the marginality of the ITL method while avoiding its volatility.
3) The allocation trend of the U-ITL is similar to the allocation trend of the ITL procedure after filtering subsidies.
4) Although the proportional sharing procedure takes into account the network, its allocation trend is similar to the allocation trend of the PR algorithm.
5) The PR method does not take into account the network and produces substantially different results than other methods.

Table VI provides a qualitative systematic comparison of the methods analyzed.

Final recommendations are as follows.

1) *Pro rata* procedures are not advisable because they are unfair for specific groups of generators and demands. Generators close to load centers are unfairly treated

TABLE VI
QUALITATIVE COMPARISON OF THE FOUR TRANSMISSION LOSS ALLOCATION PROCEDURES

	Methods compared			
Characteristics	PR	ITL	U-ITL	PS
Is it quantity dependent?	yes	yes	yes	yes
Is it network dependent?	no	yes	yes	yes
Does it depend on the slack bus?	no	yes	no	no
Does it require linearity?	yes	no	no	yes
Is it marginal?	no	yes	yes	no
Does it produce negative losses?	no	yes	no	no
Is it volatile?	no	yes	no	no
Is it easy to understand?	yes	yes	yes	yes
Is it simple to implement?	yes	yes	yes	yes

with respect to generators far away from load centers. Analogously, demands close to generating areas are unfairly treated with respect to demands far away from those areas.

2) If the slack bus is unique and volatility, negative losses and allocation imbalance are acceptable, the ITL procedure is advisable.
3) If volatility, negative losses and allocation imbalance are not desired, the U-ITL and the proportional sharing algorithms are recommended.

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