Chapter 4. Participating in Markets for Electrical Energy
Previously: we have discussed the basic principles of electricity markets.

Now: we discuss the decisions that generators, consumers and others take to optimize their benefits.

Which others? Storage facilities, hybrid participants.

Market not perfectly competitive ⇒ optimization needs to be done while taking into account the behavior of other participants.
The consumer’s perspective

If they pay a flat rate for electricity ⇒ demand only affected by the cycle of their activities. Averaged over a few months, their demand reflects their willingness to pay this flat rate.

What if the price fluctuates more rapidly? Almost no demand response because price elasticity for the demand is usually small.

Value Of Loss Load (VOLL): is the estimated amount that customers receiving electricity with firm contracts would be willing to pay to avoid a disruption in their electricity service.
Pool selling price: electricity pool of England and Wales (in £/MWh). VOLL for the same period 2768 £/MWh.

Small elasticity partially explained by the fact that VOLL much greater than average price of electricity.
Shifting demand

Shifting demand: rather than reducing their demand, the consumers may decide to delay this demand until the prices are lower. This concept exists for a long time with for example the night and day tariffs.

There exist many opportunities for shifting demand that can still be exploited, even for small customers (e.g., turning off the fridge for half an hour, delaying a laundry).

Investments in systems to exploit these shifting demand opportunities important in a landscape where more and more electricity is produced by renewables.

Investments: recording consumers’ consumption for every market period (essential for not purchasing anymore electricity on the basis of a tariff), automatic devices installed in homes for shifting loads, etc.
Retailers for electrical energy

Small consumers usually prefer purchasing on a tariff = constant price per kilowatt-hour rather than to be active participants. They buy their energy to a retailer.

Challenge: to buy energy at a variable price on the wholesale market and sell it a fixed price at the retail level.

The quantity-weighted average price at which a retailer purchases energy should be lower than the rate it charges its customers.

Must forecast very well the consumption of its consumers to reduce its exposure to spot market prices (accuracy usually good if a large group of customers).

Retailers may offer more competitive tariffs to customers which record the energy consumed at every time period.
Example

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<td>256</td>
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<td>Contract purchases</td>
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<td>3066</td>
<td>2794</td>
<td>1049</td>
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<td>1556</td>
<td>2765</td>
<td>241</td>
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Flat retail price: 38.50 $/MWh.
The producer’s perspective

We focus on a generating company that tries to maximize the profits it derives from a single generating unit called unit $i$.

Problem: $\max \Omega_i = \max [\pi P_i - C_i(P_i)]$ where $P_i =$ power produced and $C_i(P_i) =$ cost of producing $P_i$.

Optimality when $\frac{d\Omega_i}{dP_i} = \frac{d(\pi P_i)}{dP_i} - \frac{dC_i(P_i)}{dP_i} = 0$.

First term $=$ marginal revenue of unit $i$ ($MR_i$). Second term $=$ marginal cost of production of unit $i$ ($MC_i$).

For optimality: $MR_i = MC_i$. 

Basic dispatch

Competition is supposed to be perfect. Price \( \pi \) not affected by \( P_i \).

Optimality when \( \frac{dC_i(P_i)}{dP_i} = \pi \).

As long as \( \pi \) is given, scheduling of the units can be done independently.

If \( P_i \) solution greater than \( P_i^{\text{max}} \) generator \( \Rightarrow P_i = P_i^{\text{max}} \). If \( P_i \) less than \( P_i^{\text{min}} \), additional check needs to be done to be sure that the generator will not be loosing money.
Example

Consider the unit with the inverse production function (quantity of fuel needed to generate $P_1$) \( H_1(P_1) = 110 + 8.2P_1 + 0.002P_1^2 \) MJ/h with a minimum stable generation is 100 MW and a maximum output of 500 MW.

Cost of fuel \( F = 1.3 \) $/MJ.

Questions: (I) What is the power that should be generated by the unit to maximize profit if electricity can be sold at 12 $/MWh? (II) At which electricity prices should the unit operate at maximum output? (III) What is the electricity price below which the unit cannot make any profit?
(I) Cost of production if $F$ (in $$/MJ) is cost in fuel per $$/MJ:
110F + 8.2P_1F + 0.002P_1^2F \$/h.

Since $F = 1.3$ $$/MJ \Rightarrow C_1(P_1) = 143 + 10.66P_1 + 0.0026P_1^2$ $$/h.

Optimality condition: $\frac{dC_1(P_1)}{dP_1} = 10.66 + 0.0052P_1 = 12$ $$/MWh \Rightarrow P_1 = 257.7$ MW.

Solution valid because in between $P_{min}$ and $P_{max}$.

(II) $\frac{dC_i(P_i=500)}{dP_i} = price \Rightarrow price = 13.26$ $$/MWh.
Can be computed by solving the following minimization problem:

\[
\min_{\text{price}, P_1} \text{price}
\]

under the following constraints:

\[
\text{price} \times P_1 - H_1(P_1)F \geq 0
\]

\[
P_1 \geq 100
\]

\[
P_1 \leq 500
\]
Question: What is the optimal dispatch for a market price $\pi$ when having these piecewise linear cost curves?
More realistic scheduling

The production profile needs to be optimized over several market periods rather than one due to (among others):

**Start-up costs:** costs of starting units. Diesel generators and open cycle gas turbines = low start-up costs. Large thermal units: large amount of heat energy before the steam is at a temperature and pressure that are sufficient to sustain the generation of electric power. They have large start-up costs.

**Dynamic constraints:** Limits placed on the variation of production of a generator to avoid mechanical stress (mainly of the prime mover) and all the problems related to gradients in temperature.

**Environmental constraints:** E.g.: rate at which a certain pollutant is released in the atmosphere is limited (or total over one year); constraints on the use of water for hydro plants.
<table>
<thead>
<tr>
<th>Hour</th>
<th>1</th>
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<th>5</th>
<th>6</th>
<th>7</th>
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<td>Price ($/MWh)</td>
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<td>13.0</td>
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<td>10.5</td>
<td>12.5</td>
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<td>500.0</td>
<td>100.0</td>
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<td>500.0</td>
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<td>6750</td>
<td>1050</td>
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<td>Running cost ($)</td>
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<td>Total cost ($)</td>
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<td>5467</td>
<td>6123</td>
<td>1235</td>
<td>4240</td>
<td>6123</td>
<td>1933</td>
</tr>
<tr>
<td>Profit ($)</td>
<td>-571</td>
<td>383</td>
<td>627</td>
<td>-185</td>
<td>183</td>
<td>627</td>
<td>-75</td>
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<td>Cumulative profit ($)</td>
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<td>-188</td>
<td>439</td>
<td>254</td>
<td>437</td>
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<td>989</td>
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![Bar chart showing prices over the day](chart.png)
The production versus purchase decision

Suppose that a generation company has signed a contract for the supply of a given load $L$ during a single hour. How should it use its portfolio of $N$ generating plants?

**Problem:** Minimize $\sum_{i=1}^{N} C_i(P_i)$ subject to $\sum_{i=1}^{N} P_i = L$ where $P_i$ represents the production of unit $i$ of the portfolio and $C_i(P_i)$ the cost of producing this amount of power with this unit.

**Solution:** form the Lagrangian function $l$, compute the values of the variable that sets its partial derivatives equal to zero to get necessary conditions for optimality.

$$l(P_1, P_2, \ldots, P_N, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda(L - \sum_{i=1}^{N} P_i)$$ where $\lambda$ is the Lagrangian multiplier.
These conditions can be written:

\[ \frac{\partial l}{\partial P_i} = \frac{dC_i}{dP_i} - \lambda = 0 \quad \forall i = 1, \ldots, N \]

\[ \frac{\partial l}{\partial \lambda} = (L - \sum_{i=1}^{N} P_i) = 0 \]

From there: \[ \frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \ldots = \frac{dC_N}{dP_N} = \lambda. \]

Lagrange multiplier is thus equal to the cost of producing one additional megawatt-hour with any of the generating units ⇒ often called the shadow price of electricity.

If market price \( \pi \) lower than shadow price \( \lambda \), the company should buy electricity on the market (decrease \( L \)) up to the point at which:

\[ \frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \ldots = \frac{dC_N}{dP_N} = \pi \]
Imperfect competition

It is quite common for an electricity market to consist of a few strategic players and a number of price takers.

The few strategic players may play on the fact that they influence the price. Often they own several units. The price $\pi$ is no longer a variable on which the firm cannot act.

The total profit of a firm $f$ that owns multiple generating units is $\Omega_f = \pi \cdot P_f - C_f(P_f)$ where:

(i) $P_f = \text{total output}$
(ii) $C_f(P_f) = \text{minimum cost for producing } P_f.$

$\Omega_f$ does not depend anymore here only on $P_f$!
The Nash equilibrium

Let $\Omega_f = \Omega_f(X_f, X_{-f})$ where $X_f$ represents the actions (called also the strategic variables) of firm $f$ and $X_{-f}$ those of its competitors.

If other firms behave in a rationale way, it is “reasonable” for each firm $f$ to select the strategic variable $X_f^*$ such that:

$$\Omega_f(X_f^*, X_{-f}^*) \geq \Omega_f(X_f, X_{-f}^*) \quad \forall X_f \forall f$$

where $X_{-f}^*$ represents the optimal action of the other firms.

The strategic profile $(X_f^*, X_{-f}^*)$ is the Nash equilibrium of a noncooperative game.

A model of strategic interaction is required for computing a Nash equilibrium.
The Cournot model

Cournot model: a model of strategic interaction where the quantities $P_f$ are the strategic variables.

What are the elements required for computing a cournot equilibrium?

[1] The strategic space of each firm (set of values for $P_f$)
[2] The production cost function $C_f(P_f)$ of each firm
[3] The inverse demand curve $\pi(P)$

If $n$ firms and if each strategic space has $m$ elements $\Rightarrow$ the condition $\Omega_f(X^*_f, X^*_{-f}) \geq \Omega_f(X_f, X^*_{-f})$ $\forall X_f \forall f$ needs to checked for $m^n$ strategic profiles to find all Nash equilibria.
Example of computation of a Cournot equilibrium

We consider the case of two firms (A and B) that compete for the supply of electricity.

[1] Strategic space for each firm: \{5,10,15,20,25,30,35\} MW
[2] \(C_A = 35P_A\) $/h, \(C_B = 35P_B\) $/h
[3] Inverse demand function \(\pi = 100 - (P_A + P_B)\) $/h
<table>
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<td>45 60 70 80 90 100 110 120 130 140 150 160 170 180</td>
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*Production of firm A*
A more analytical approach

Profits firm A: $\Omega_A(P_A, P_B) = \pi(P_A + P_B) \cdot P_A - C_A(P_A)$

Profits firm B: $\Omega_B(P_A, P_B) = \pi(P_A + P_B) \cdot P_B - C_B(P_B)$

For each of these problems we can write a condition of optimality:

$$\frac{\partial \Omega_A}{\partial P_A} = \pi(P_A + P_B) - \frac{dC_A(P_A)}{dP_A} + P_A \cdot \frac{d\pi}{dP} \cdot \frac{dP}{dP_A} = 0$$

$$\frac{\partial \Omega_A}{\partial P_B} = \pi(P_A + P_B) - \frac{dC_B(P_B)}{dP_B} + P_B \cdot \frac{d\pi}{dP} \cdot \frac{dP}{dP_B} = 0.$$  

where $P = P_A + P_B$.

This gives the reaction curves: $P_A = \frac{1}{2}(65 - P_B)$ and $P_B = \frac{1}{2}(55 - P_A)$. Solving these equations gives the same equilibrium as before: $P_A = 25$ MWh, $P_B = 15$ MWh and $\pi = 60$ $$/\text{MWh}.$$
Plants with very low marginal costs

Several types of plants (nuclear, hydroelectric, renewable) have (almost) negligible marginal costs ⇒ challenge is to recover the investments.

Nuclear units: must operate at an almost constant generation level. Owners must sell the nominal power of their units at every hour and almost at any price.

Hydro plants (with substantial reservoir): can adjust their production; must forecast the periods when the price for electricity will be the highest and sell during these periods.

Wind farms and solar plants: production uncontrollable; require accurate prediction techniques to be able to sell their production at (not too) unfavorable prices.
The Hybrid Participant’s Perspective

Hybrid Participant: behaves like producers and consumers depending on the circumstances.

Pumped hydro plan most common type of hybrid participant.

Operation profitable if the revenue of selling energy during periods of high prices is larger than the cost of the energy consumed during periods of low prices.

Calculation needs to take into account losses! Around only 75% of the energy consumed can be sold back to the market for a pumped hydro plant.
Homework

For a group of students, explain what a load aggregator is. Present a company which is acting as load aggregator and its business model.

For a group students, present the concept of smart meter and the impact of smart meters on the electrical sector as a whole.

For a group of students, present a methodology published in a recent research paper (less than 5 years old) for forecasting loads.

For a group of students, present a methodology published in a recent research paper (less than 5 years old) for forecasting spot prices.

The presentations should not last more than 20 minutes.