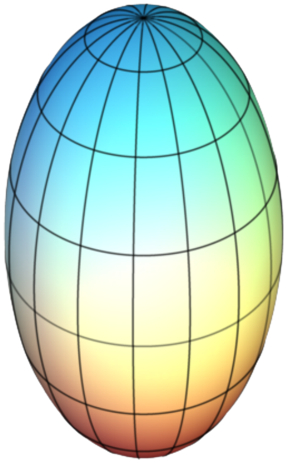
14. Tide

Low and high tide at the Bay of Fundy (Maine, USA):

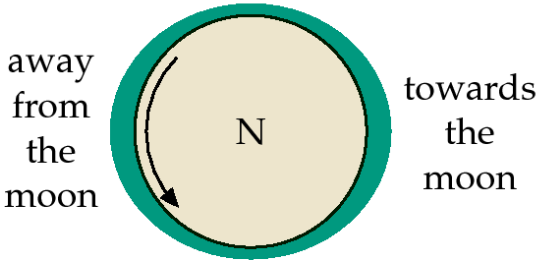
The moon and the earth move around each other and around the sun $\Rightarrow$ TIDE

A simplified model for explaining tides

Colin Maclaurin (1698-1746), a Scottish mathematician, showed that a smooth sphere covered by a sufficiently deep ocean under the tidal force of a single deforming body is a prolate spheroid.

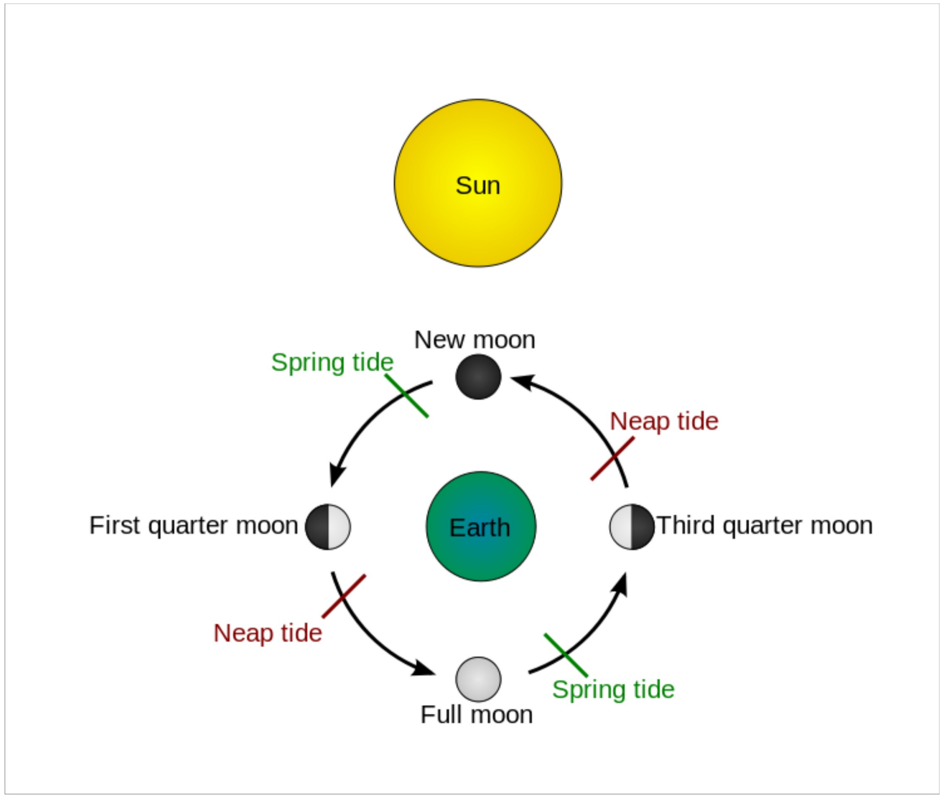


If we assume that **(i)** we can neglect the rotation of the moon around the earth and the effect of the Sun, **(ii)** the earth can be modeled by MacLaurin's sphere, **(iii)** it is relevant to use a quasi-static model for ocean dynamics $\Rightarrow$ We have an explanation for why there are two high tides and two low tides per day.



(i-A) Effect of the rotation of the moon around the earth. The moon orbits once every 28 days around the earth $\Rightarrow$ the tidal period is not 12 hours but 12 hours and 25.2 minutes.

(i-B) Effect of the sun on the tides. The effect of the moon on tides is much bigger than the effect of the sun (which is much further from the earth). At full moon and at new moon, the effects of the sun and the moon reinforce each other, and the resulting big tides are called spring tides. At the intervening half-moons, the imbalances cancel out and the tides are smaller.



(ii) The earth cannot be modeled by a MacLaurin's sphere. Continents get in the way of tidal waves. The true behavior of tides is more complicated. In the Atlantic Ocean, tidal crests and troughs form but are unable to move around the earth; instead they more around a perimeter of the ocean in an anticlockwise direction; this takes about 25 hours.

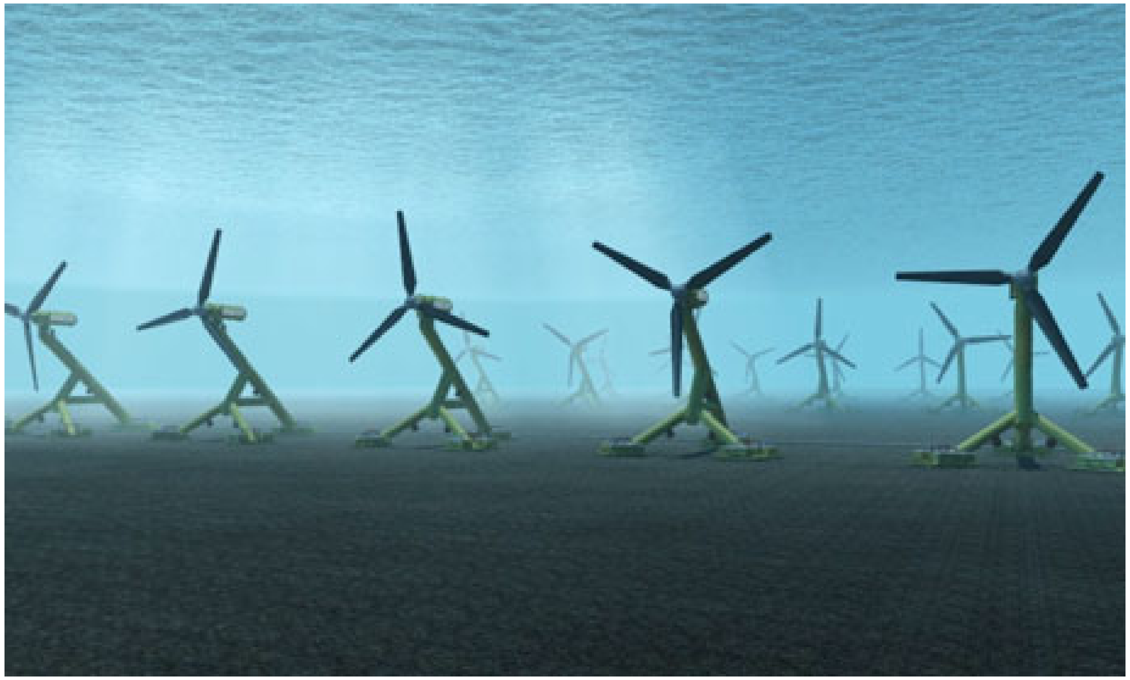
(iii¡) What about the limitations of the quasi-static model? It cannot explain the dynamics of the propagation of the tidal waves. The quasi-static model also ignores the Corriolis force that plays an important role and explains, for example, why tides in the English channel are bigger on the French side.

Raw incoming tidal power

Tidal waves arrive from the Atlantic and move at a speed of around 70 km/h. Power that can be extracted from tides can never be larger than the total power from these waves, which have an average power of 250 GW, that's around $\frac{250\times10^9\times24}{60\times10^6\times1000} \simeq 100$ kWh per day per person.

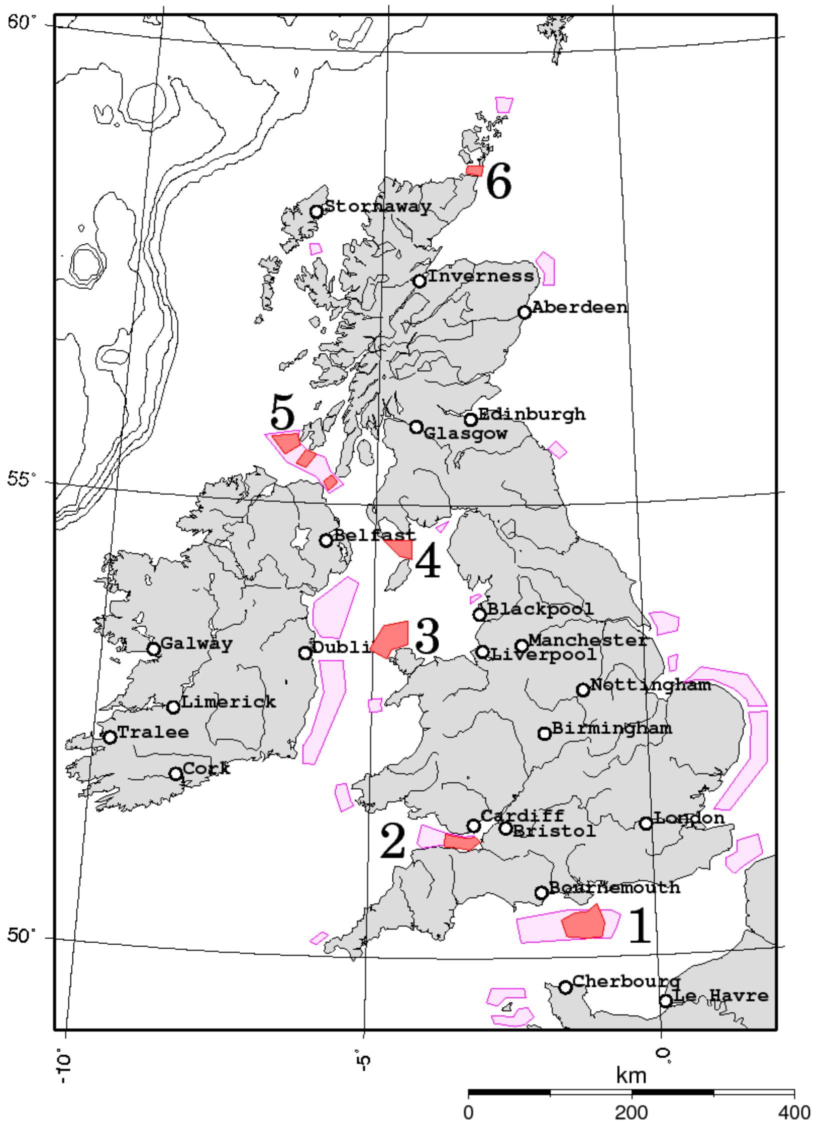
There are two technologies for extracting power from these tidal waves: **tidal farms** and **tidal pools**. Tidal pools can be created either by building **barrages** or **lagoons**.

Tidal stream farms



These extract energy in the same way wind farms do.

If we assume that the same rules apply to tidal stream farms as wind farms, we have that the power per unit sea-floor area is: $\frac{\pi}{200}\frac{1}{2}\rho\mathcal{U}^3$ where ρ is the density of water and $\mathcal{U}$ the speed of tidal currents.

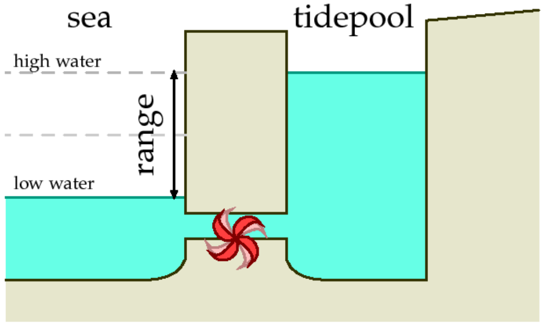


Regions around the British Isles where peak tidal flows exceed 1 m/s. Red regions have water depths greater than 100 m.

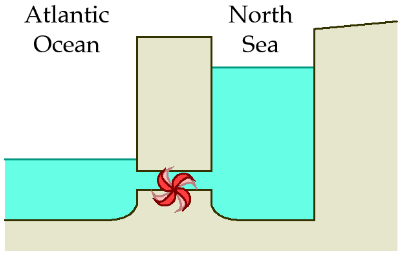
If tidal farms were to be placed in these areas, an estimated 9 kWh/d per person could be generated.

Tidal pool

A tidal pool is an **artificial pool next to the sea**, with a water-wheel/hydraulic generator that is turned as the pool fills or empties. The pool is filled at high tide and is emptied at low tide.

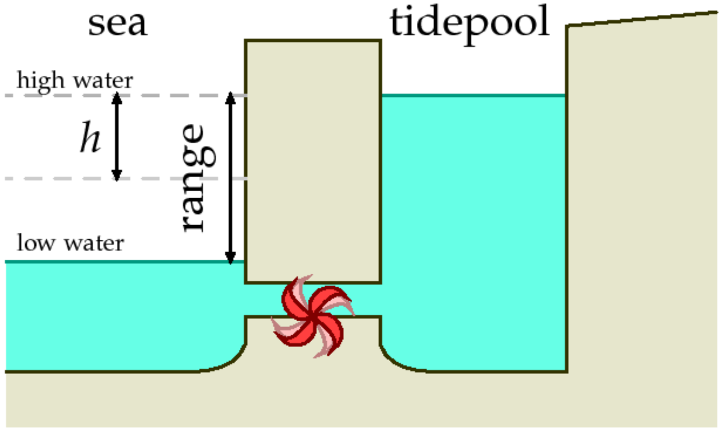


A superb and ambitious project would be to transform the North Sea into a water pool:



Estimating the amount of power a tidal pool can deliver

Let us estimate the power (per m^2) of an artificial tide pool that's filled rapidly at high tide and emptied rapidly at low tide, generating power from both flow direction. Let $h = \frac{range}{2}$ and ϵ the efficiency of the hydraulic generator



Phase A. The pool is at height range and there is low water in the sea. Mass of water per $m^2 = 2\rho h$. The change in potential energy of the water per m^2 of tidal pool when flowing from the tidal pool to the sea: $2\rho hgh$.

Phase B. The pool is at height 0 and there is high water in the sea. The change in the water's energy when flowing from the sea to the tidal pool: $2\rho hgh$.

Two phases A and B per day $\Rightarrow$ Potential energy per unit area of a tidal pool that can be exploited per day to generate electricity: $8\rho hgh$.

We therefore have a power per unit area of tidal pool equal to:

$$\frac{8\rho \times h^2 \times g \times \epsilon}{3600 \times 24}$$

By assuming a range equal to 4 m (i.e., $h = 2$ m) for tides and an efficiency $\epsilon = 0.9$, we have a power per unit area of tidal pool $\frac{16\rho \times h^2 \times g \times 0.9}{3600 \times 24} \simeq$ 3.3 W/m^2.

Since the area of the North Sea is 750,000 km^2, by transforming it into a tidal pool, we could extract $\frac{3.3 \times 750{,}000 \times 10^6 \times 24}{60 \times 10^6 \times 1000} \simeq$ **990 kWh/d per person ?!???!!!!????** $\Rightarrow$ incorrect assumptions. The pool model does not apply to the North Sea for several reasons: $h = 2$ m is what is observed in estuaries but not everywhere in the North Sea, the size of the North Sea is not negligible with respect to the length of the tidal waves, etc.

Tidal pools with pumping

The pumping trick artificially increases the amplitude of the tides in a tidal pool so as to amplify the power obtained. At high tide, water is pumped into the tidal pool until a height $b + 2 \times h$ is reached. The energy cost of pumping in extra water at high tide is repaid with interest when the same water is let out at low tide.

Let us assume that generation has an efficiency of ϵ_g and that pumping has an efficiency of ϵ_p. The energy cost of pumping water per m^2 of tidal pool to boost height by b m is $\frac{b^2 \times \rho \times g}{2 \times \epsilon_p}$ and the additional energy generated at low tide is $\epsilon_g \times (2h + \frac{b}{2}) \times b \times \rho \times g$. By defining the round trip efficiency of $\epsilon = \epsilon_g \epsilon_p$, the optimal extra height is $b = 2h \frac{\epsilon}{1-\epsilon}$.

For example, with a tidal range of $2h = 4$ m and a round-trip efficiency of $\epsilon = 75\%$, the optimal boost is around 13 m.

Barrages for tidal pools

Concept: Building a barrage in an estuary where tides have a high range to form a tidal pool. Proven technology.

Picture: Tidal barrage at La Rance (France). Average tidal range is 8 metres. The barrage has been producing an average power of 60 MW since 1966.

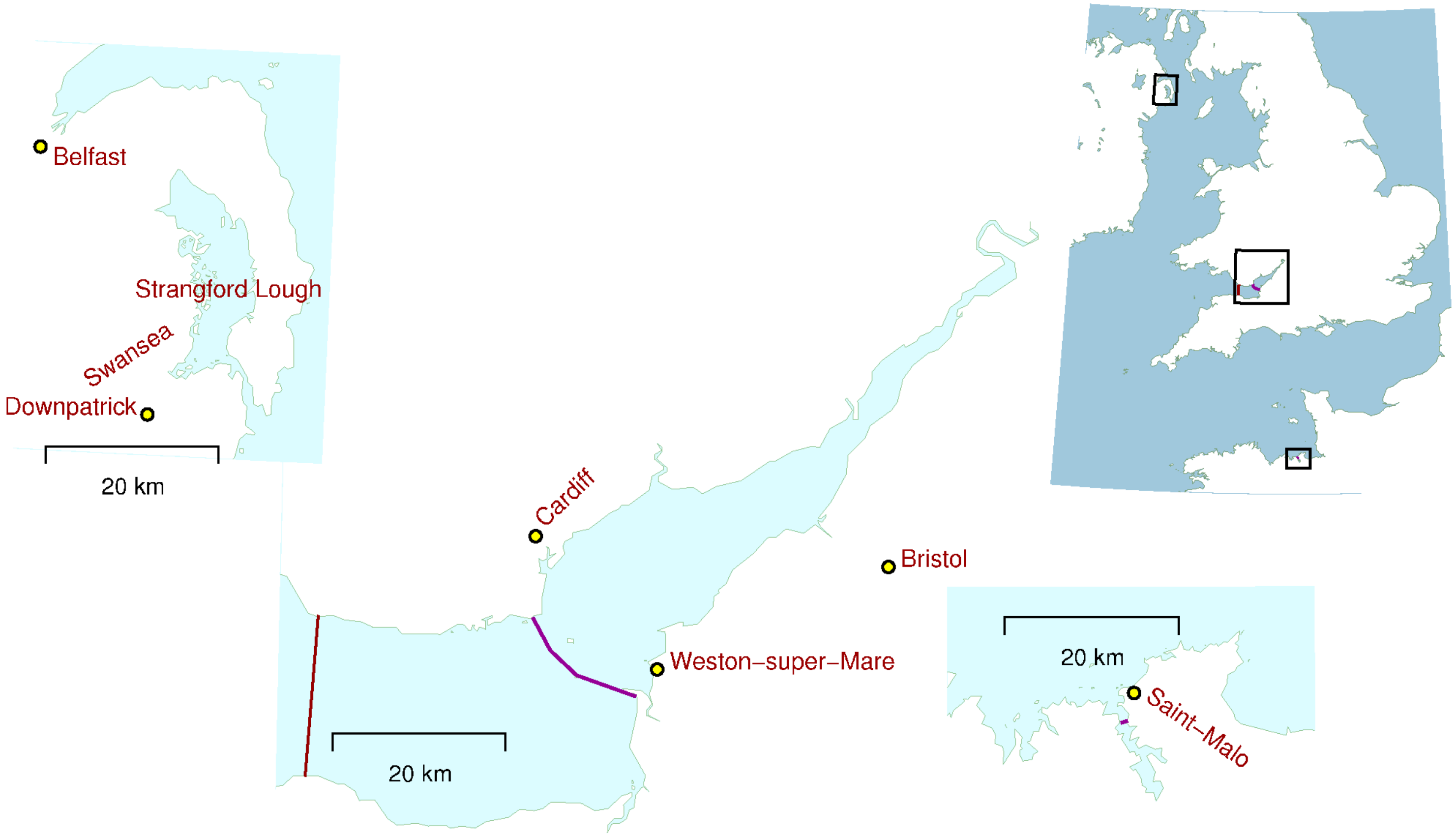


Figure: The Severn barrage proposals (bottom left), and Strangford Lough, Northern Ireland (top left), shown at the same scale as La Rance.

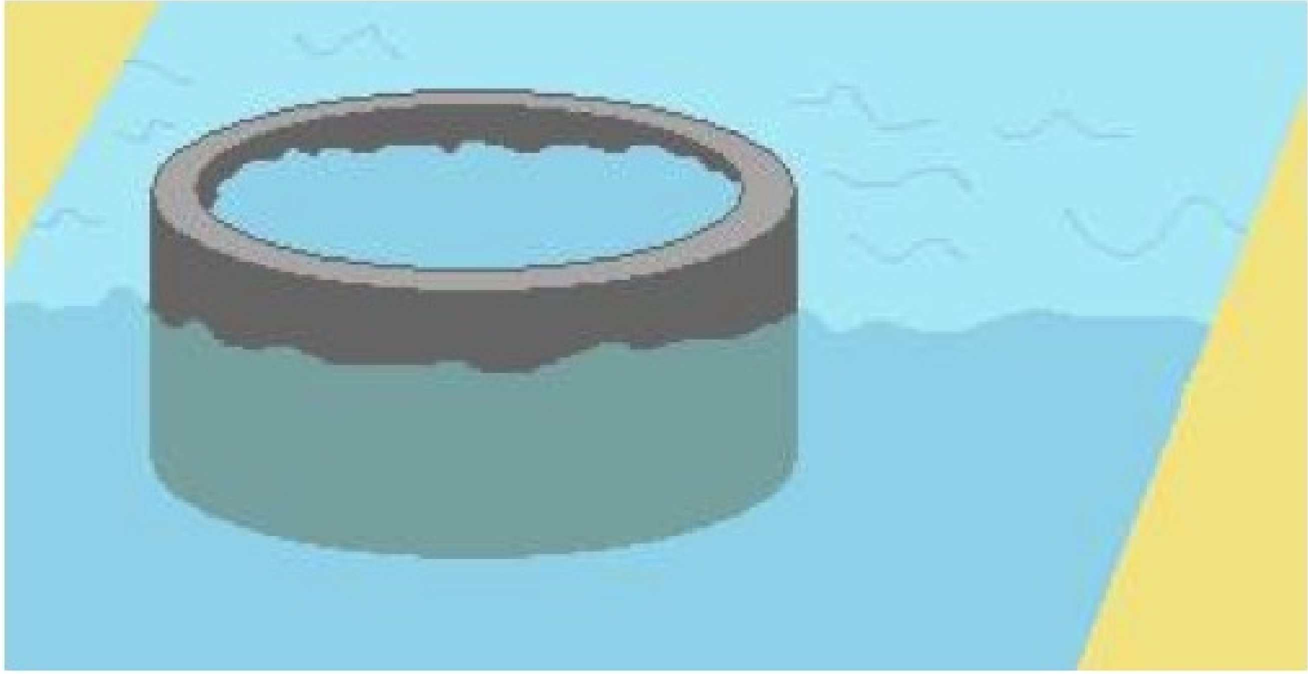
The Severn barrage

Data: Tidal range at Cardiff is 11.3 m at spring tides and 5.8 at neap tides. The size of the tidal pool formed by a barrage going from Weston-super-Mare to Cardiff: 500 km^2.

Using formula $\frac{8\rho \times h^2 \times g \times \epsilon}{3600 \times 24}$ and choosing $\epsilon = 0.9$, this tidal pool could produce could produce 26.6 W/m^2 at spring tides and 7 W/m^2 at neap tides. That would be equivalent to **10.6 kWh/d per person** and **1.4 kWh/d per person**, respectively.

These numbers assume that the water is let in at single pulse at the peak of high tide, and let out at single pulse at low tide. In practice, in-flow and out-flow are spread over several hours $\Rightarrow$ reduction in power delivered. Current proposals for the barrage will generate power in one direction $\Rightarrow$ reduces the power by another 50 %. Engineers report that current proposals would contribute around 0.81 kWh/d per person on average.

Tidal lagoons: an alternative to tidal barrages



Tidal lagoons utilize a circular (or more organically shaped) retaining wall to hold high or low tides in a central pool, before releasing them to drive turbines. Two conditions are required for building lagoons: the water must be shallow and the tidal range must be large.

There are two locations for tidal lagoons in Britain: The Wash on the east coast and the waters off Blackpool on the West coast. Each has an area of 400 km^2. By assuming an average power of 4.5 W/m^2, the power generated by these lagoons could be 1.5 kWh/d per day.

Tidal stream farms (9 kWh/d per person) + the barrage (0.8 kWh/d per person) + lagoons (1.5 kWh/d per person) $\simeq$ 11 kWh/d per person.

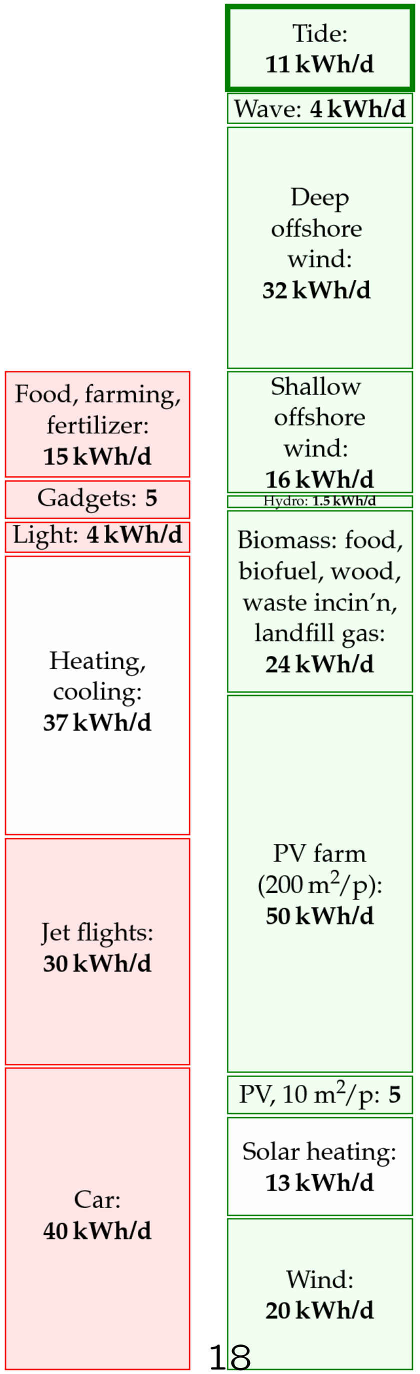
Several other reasons for being excited about tidal power:

1. Tidal power is completely predictable, unlike sun or wind power.

2. More constant than the sun or the wind.

3. Relatively cheap.

4. Humans live on land and not on the sea (or under the sea), so, they won't object to lagoons or tidal farms.



But what would you answer to people who say:

Extracting power from the tides slows down the earth's rotation so it cannot be called renewable and it may even lead to a **disaster in the long-run** (if the earth stops spinning).

Answer: Natural tidal friction already slows down the earth's rotation by 2.3 milliseconds per century. The natural rotational energy loss is roughly 3 TW (10^{12} W or $\frac{10^{12}\times 24}{60\times 10^{6}\times 10^{3}} \simeq$ 400 kWh/d per person). Many tidal energy extraction systems are just extracting energy that would have been lost anyway in friction. But even if we doubled or tripled the power extracted from the earth-moon system, tidal energy would still last hundreds of millions of years.

15. Stuff

It is conventional to divide the energy-cost of manufacture into four phases:

Phase R: Making raw materials. Digging minerals out of the ground, melting them, purifying them, and modifying them into manufacturers' lego (e.g., plastics, glasses, metals and ceramics). Energy costs in this phase include the transportation of the raw materials.
Phase P: Production. Raw materials are processed into a manufactured objects. Energy costs of this phase include packaging and more transportation.
Phase U: Use. Energy consumed by goods when in use (e.g., a hair-dryer uses electricity).
Phase D: Disposal. This includes the energy cost of putting goods in a landfill, of turning them back into raw materials (recycling) and of cleaning all the pollution associated with them.

Remarks: (i) Usually the total energy cost of one phase dominates the others. (ii) Costs of phases **R** and **P** are called the **embodied** or **embedded** energy (iii) We will just focus on the phases R and P of a few common items: drink containers, computers, junk mail, cars and houses

Drink containers. Making one aluminium drinks-can uses 0.6 kWh. For a 500 ml plastic bottle, we need 0.7 kWh. So, a five-a-day habit wastes 3 kWh/day.

Other packaging. An average UK inhabitant throws away 400 g of packaging per day, mainly food packaging. The embodied energy in packaging is on average around 10 kWh/kg ⇒ The energy footprint of packaging is 4 kWh/d.

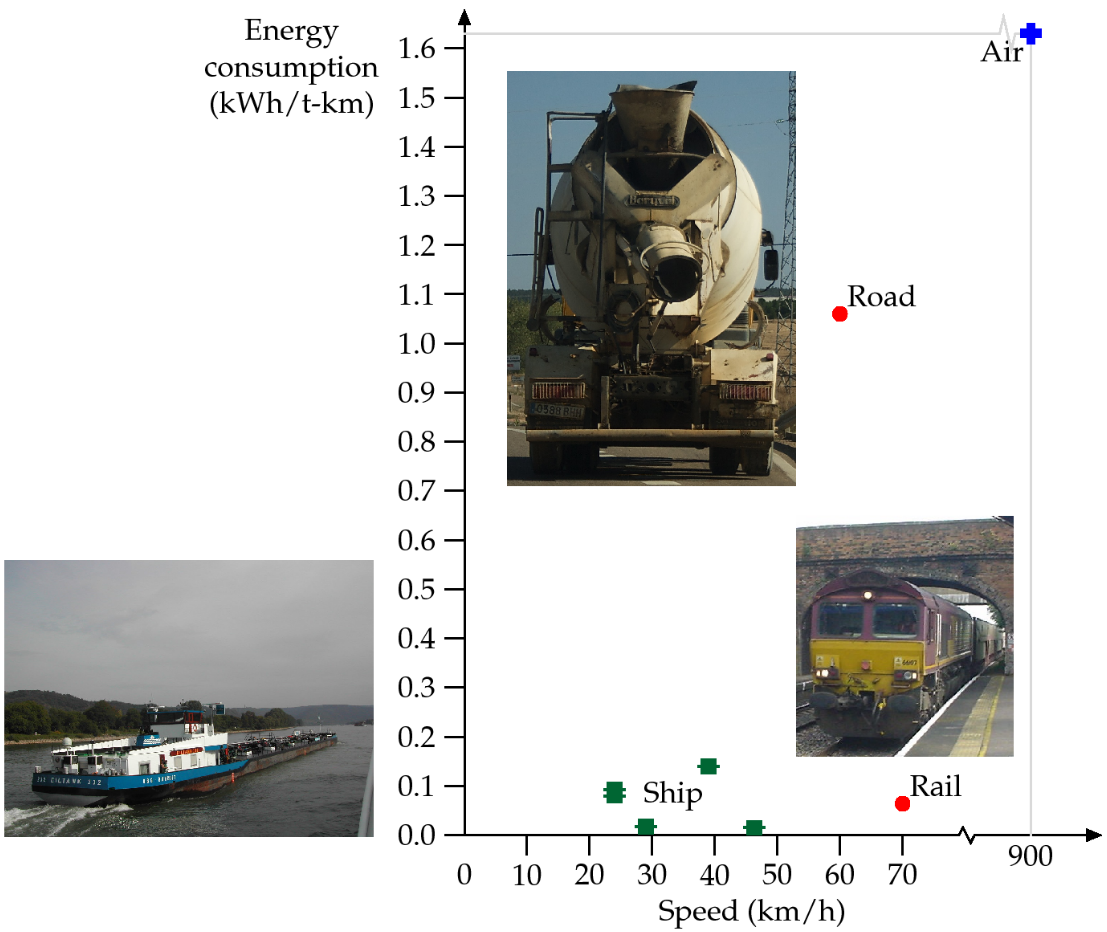
Computers. Energy cost per computer ≃ 1800 kWh. Once computer every two years ⇒ 2.5 kWh per day.

Newspapers, magazines, and junk mail. 3 grams per page of newspaper. Paper has an embodied energy of 10 kWh per kg. The typical flow of junk mail, magazines, and newspapers amounts to 200 g of paper per day which is about 2 kWh per day.

Bigger stuff. Houses: Assuming that we replace a house after 100 years, the energy cost is 2.3 kWh/d. Knowing that the average occupancy of a house is 2.3, the average energy expenditure on house-building is 1 kWh per day per person.
Cars: A car has an embodied energy of 76,000 kWh. Assuming it lasts 15 years, that's an energy cost of 14 kWh per day.
Roads: Cost of 7600 kWh per meter plus a maintenance cost over 40 years equal to 37400 kWh. In the UK, there are 42000 km of roads. So those roads cost 2 kWh/d per person.

Transporting goods

Freight-transport is measured in ton-kilometers.



Energy of road transportation in the UK is around **1 kWh per t-km**. For a container ship it is around **0.015 kWh per t-km**.

Transporting goods by road. In 2006, the total amount of road transport in Britain by trucks was 156 billion t-km. That's 7 t-km per person, which costs 7 kWh per day per person.

Transporting goods by water. It has been estimated that Britain's share of the energy costs of international shipping is 4 kWh/day per person.

Transport of water. 162 liters used per day per person. The cost of pumping water around and treating sewage is about 0.4 kWh per day per person.

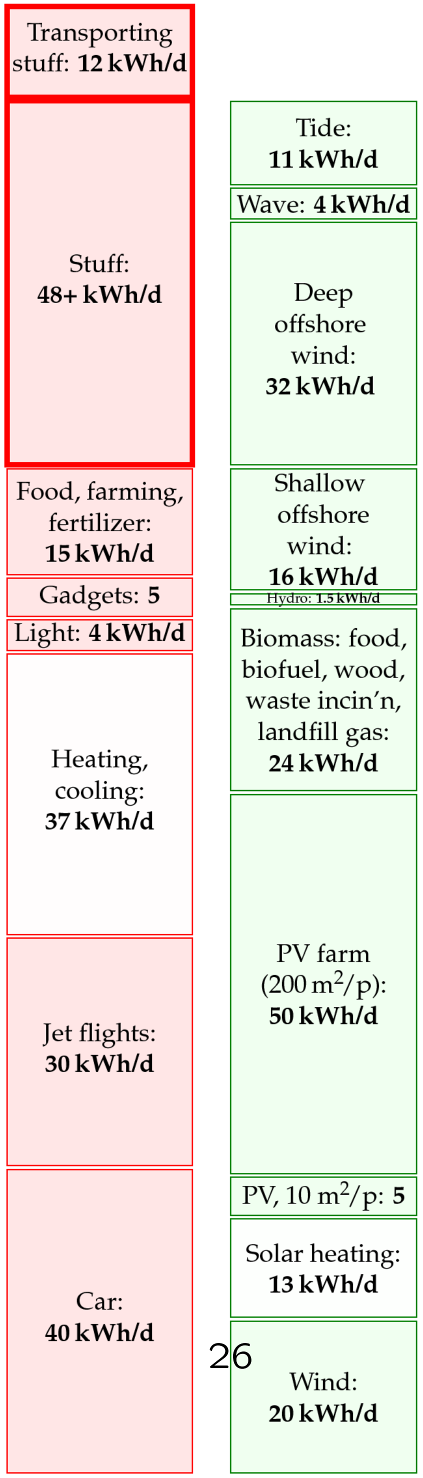
Retail

Supermarkets in the UK consume 11 TWh of energy per year, that's $\frac{11\times10^{12}}{365\times1000\times60\times10^{6}} =$ 0.5 kWh per day per person.

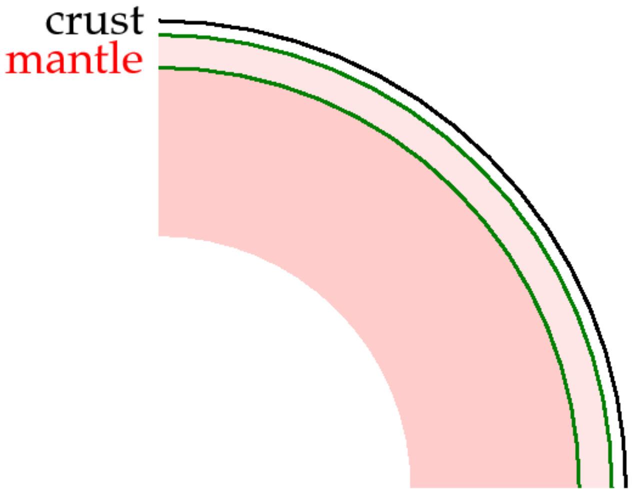
To come up with a final estimation, **(i)** we will assume that all manufactured goods (e.g. vehicles, machinery, white goods, and electrical and electronic equipment) are manufactured abroad, **(ii)** we will use an official estimate of 1.3 tons per person per year of imported manufactured goods and **(iii)** we will consider that the energy of production of manufactured goods is around 10 kWh per kg ⇒ Manufactured goods cost 40 kWh per day per person.

To this number we add 2 kWh/d per person for newspapers, 2 for roadmaking, 1 for house-making, 3 for packaging and 12 kWh per day per person for the transportation of goods by sea, by road and by pipe and storage in supermarkets.

Total: 60 kWh/d per person



16. Geothermal



Geothermal energy comes from two mains sources: (I) radioactive decay in the crust of the earth and (II) heat trickling through the mantle from the earth's core.

Where does the heat in the earth's core come from? [A] The earth used to be red hot and is still cooling down [B] the earth flexes in response to the gravitational fields of the moon and sun and this creates heat.

The Nesjavellir geothermal plant - Iceland

The plant produces 120 MW of electricity and 1100 liters of water at $\simeq$ 82°C per second. Assuming that regular water is otherwise pumped at 10 10°C, that's $1100 \times 4184 \times (82 - 10) \simeq 33$ MW of heat. As Iceland has 320,000 inhabitants, the plant produces $\frac{(120+33)\times 10^6 \times 24}{10^3 \times 320{,}000} =$ 11.4 kWh/d per person. Geothermal plants in Iceland produce around 300 MW of electricity.

How much geothermal energy is available?

We can estimate geothermal power of two types: [A] the power available at an ordinary location on the earth's crust and [B] the power available in special hot spots like Iceland. We will assume that the greater total resource comes from ordinary locations, since ordinary locations are so much more numerous.

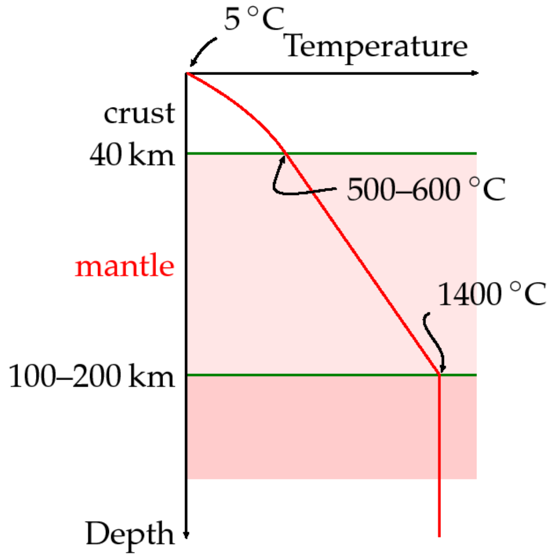
Main problem for making **sustainable** geothermal power: the speed at which heat travels through solid rock limits the rate at which heat can be sustainably sucked out of the red-hot interior of the earth. If too many "straws" suck heat at the same place, the rock may become colder and colder.

Geothermal power that would be sustainable forever

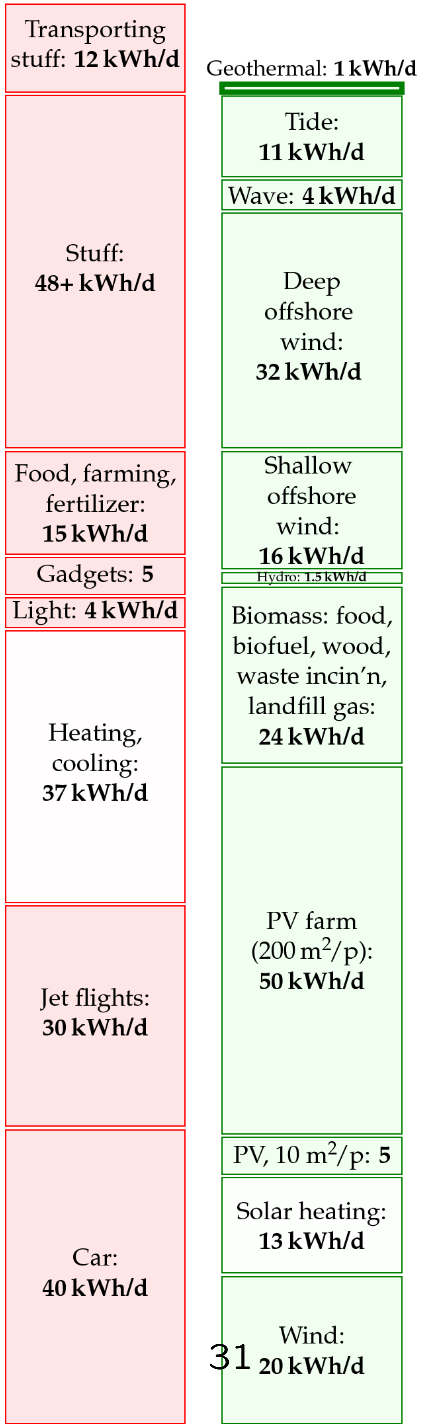
Condition for having geothermal power which is sustainable: sucking energy at a rate which is equal at most to the natural rate at which heat is already flowing out of the earth.

The rate of energy depends on the depth. The heat flow from the center coming through the mantle is about 10 mW/m^2. The heat flow at the surface is 50 mW/m^2 $\Rightarrow$ Maximum rate of energy we can get per unit area is **50 mW/m^2**.

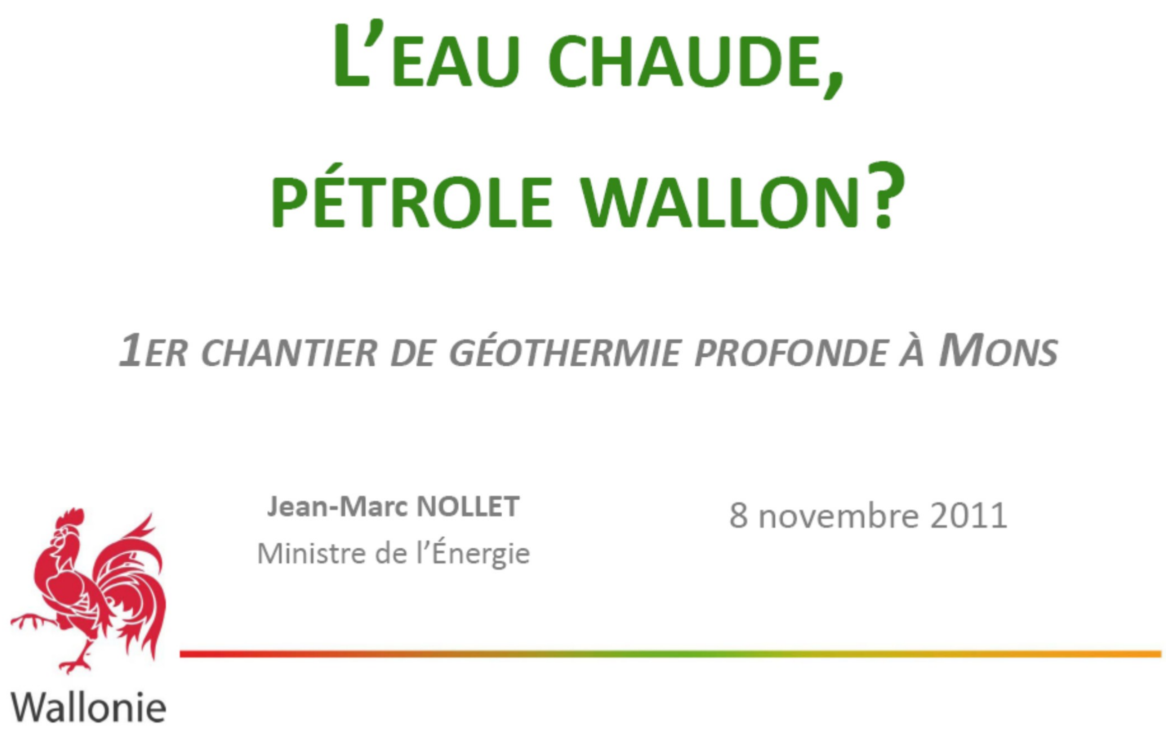
Additional problem: close to the surface the temperature is too low for producing heat that can be used for something really useful such as heating buildings or producing electricity (remember Carnot's rule that specifies that the limit on the maximum efficiency of any heat engine is $1 - \frac{T_{cold}}{T_{hot}}$ where T_{cold} is the absolute temperature of the cold reservoir and T_{hot} is the absolute temperature of the hot reservoir).



To come up with an estimate of geothermal power, we will assume that only half of the maximum power per m^2 can be turned into useful power, that's 25 mW/m^2. We will also assume that only half of the land is exploited for geothermal power $\Rightarrow$ sustainable geothermal power could only offer $\frac{0.025 \times \frac{4000}{2} \times 24}{1000} \simeq$ 1 kWh/d per person.



What do you think about this first slide of a presentation given by the Minister Jean-Marc Nollet on geothermal power in Wallonia?



Elements of answer:

1. He associates geothermal energy with oil to emphasize that geothermal energy has real potential to enrich Wallonia just as oil has enriched for Saudi Arabia.

2. Wallonia is not a hot spot for geothermal power as Iceland is. So, if geothermal power was to become vastly used in Wallonia, it could only be for a short period of time. It would be geothermal power as mining.

3. He is right to associate geothermal power with oil in the sense that geothermal resources in Wallonia will not last longer than oil if they are as intensively exploited.

17. Public services

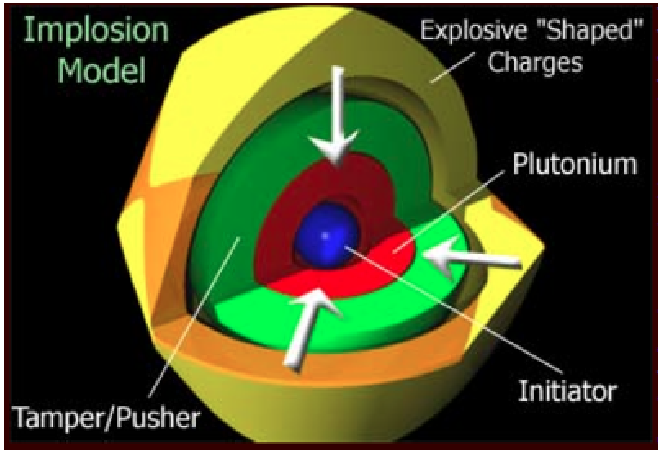
The cost of defense.

The UK's defense budget is around 6% of GDP. GDP is £587 billions. We will assume that 6% of the budget of defense is spent on energy and that the price of energy is £0.027 per kWh (Note: 6% is the fraction of GDP which is spent on energy and £0.027 is the average price of energy ⇒ Energy spent in defense is $\frac{0.06 \times 587 \times 10^9 \times 0.06}{0.027} = 80 \times 10^9$ kWh = 80 TWh. In our favorite unit that's $\frac{80 \times 10^9}{60 \times 10^6 \times 365} \simeq$ 4 kWh per day per person.

The cost of nuclear defense

The money spent by the USA on manufacturing and deploying nuclear weapons from 1945 to 1996 was \$5.5 trillion of dollars (in 1996 dollars). Assuming that (i) 6% of this expenditure went on energy (ii) an energy cost of \$0.05 per kWh, and that the US population at this times was 250 million, that's a cost of $\frac{5.5\times10^{12}\times0.06}{0.05\times250\times10^{6}\times51\times365} \simeq$ 1.4 kWh per day per American for 51 years.

Half of this energy was spent on producing nuclear materials for bombs. One gram of plutonium requires 24 000 kWh of energy. The USA has produced 104 tons of plutonium between 1945 and 1996 $\Rightarrow$ 0.5 kWh per day per person was spent during this period just to produce plutonium!



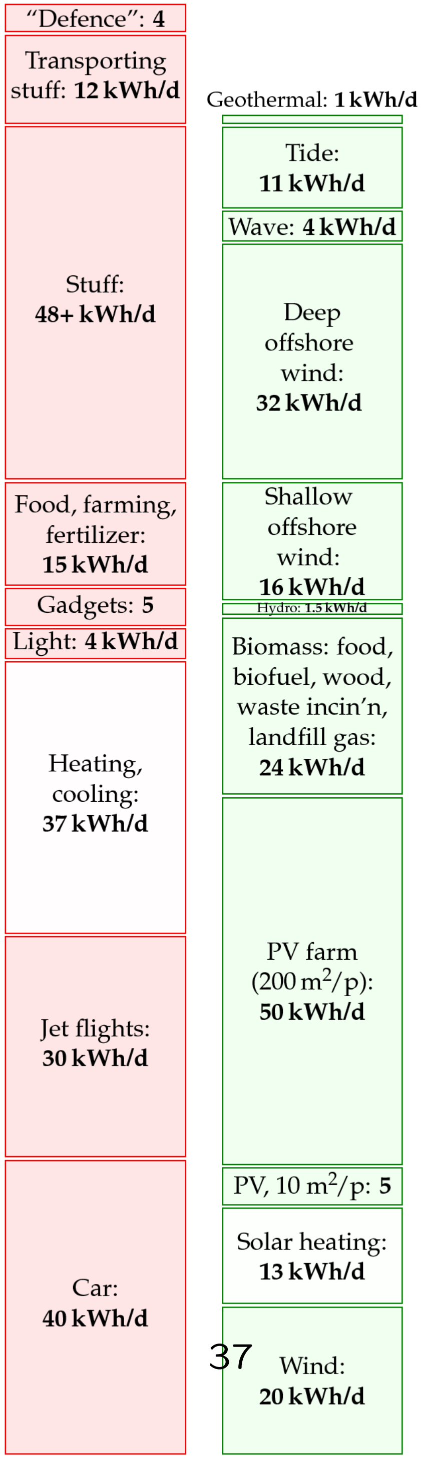
Explosion of the Tsar bomb

The largest bomb ever exploded. Energy released: 50 million tons of TNT. Energy in TNT: 4.184 MJ/kg. Exploding such a bomb every year would release $\frac{50\times10^6\times1000\times4.184\times10^6}{365\times60\times10^6\times1000\times3600} \simeq$ **2.65 kWh/d per person**.

Universities

UK universities use 5.2 billion kWh per year. Shared out among the whole population, that is 0.24 kWh per day per person.

We should probably talk about other public services but it is time now to wrap up our race between the red and the green stacks.



18. Can we live on renewables?

Several issues are raised by the green and red stacks that have been built. For example, even if our red consumption stack was lower than the green one, it would not necessarily mean that our energy sums would add up. Indeed, for a sustainable energy plan to add up, we need both the forms and amounts of energy consumption and production to match up.

The questions we address here are:

1. Is the size of the red stack roughly correct?

2. Have we been unfair to renewables by underestimating their potential?

3. What happens to the green stack when we take into account social and economic constraints?

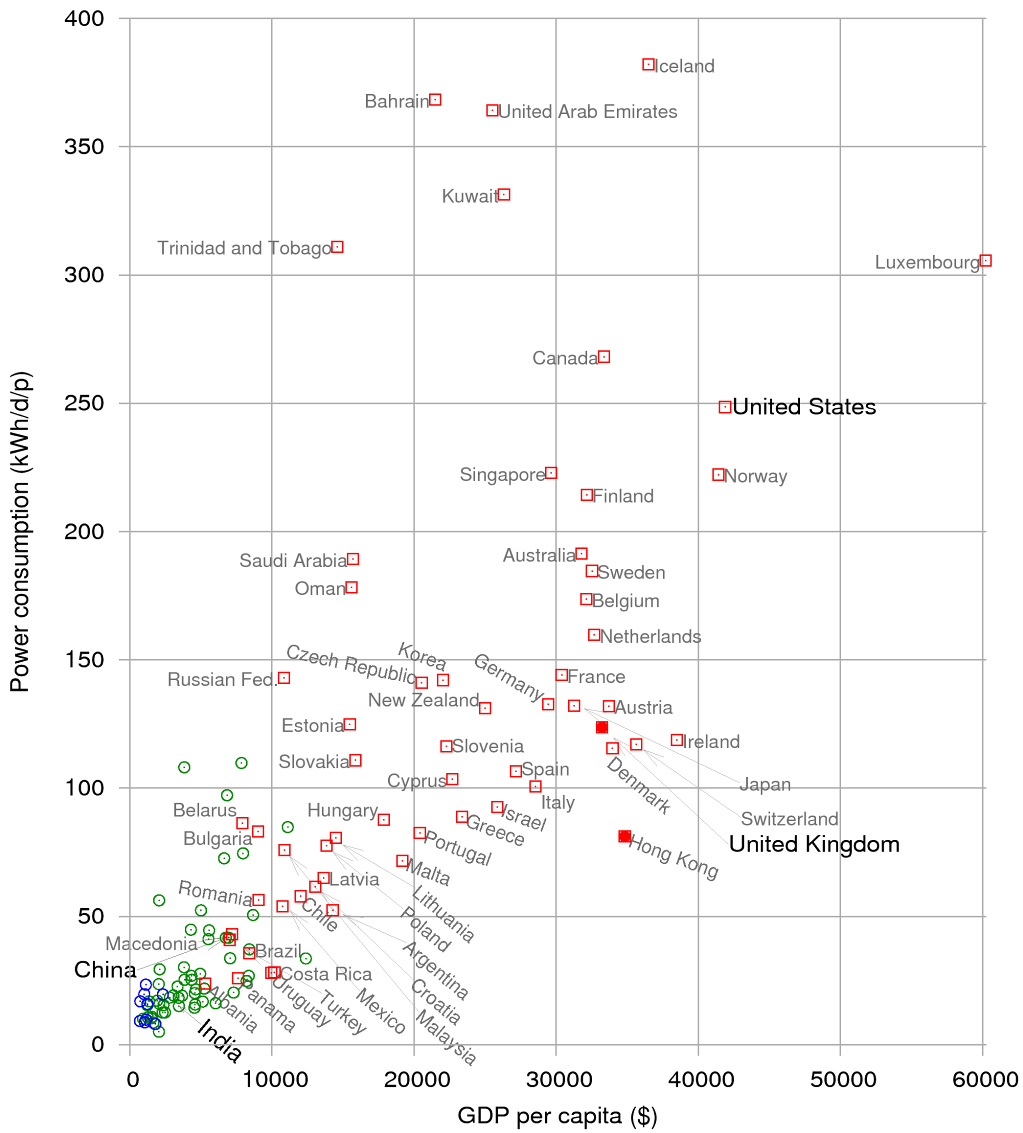
Red reflections

Official statistics for the consumption of primary energy (which means the energy contained in raw fuels, plus wind and electricity) is 125 kWh/d per person in the UK. And ... 250 kWh/d per person in the US.

These official statistics do not include the energy imported through "stuff" (40 kWh/d per person) as well as the energy embodied in food ($\simeq$ 15 kWh/d per person). On the other hand we did not include the energy lost in converting energy, which accounts for 22% of the national total.

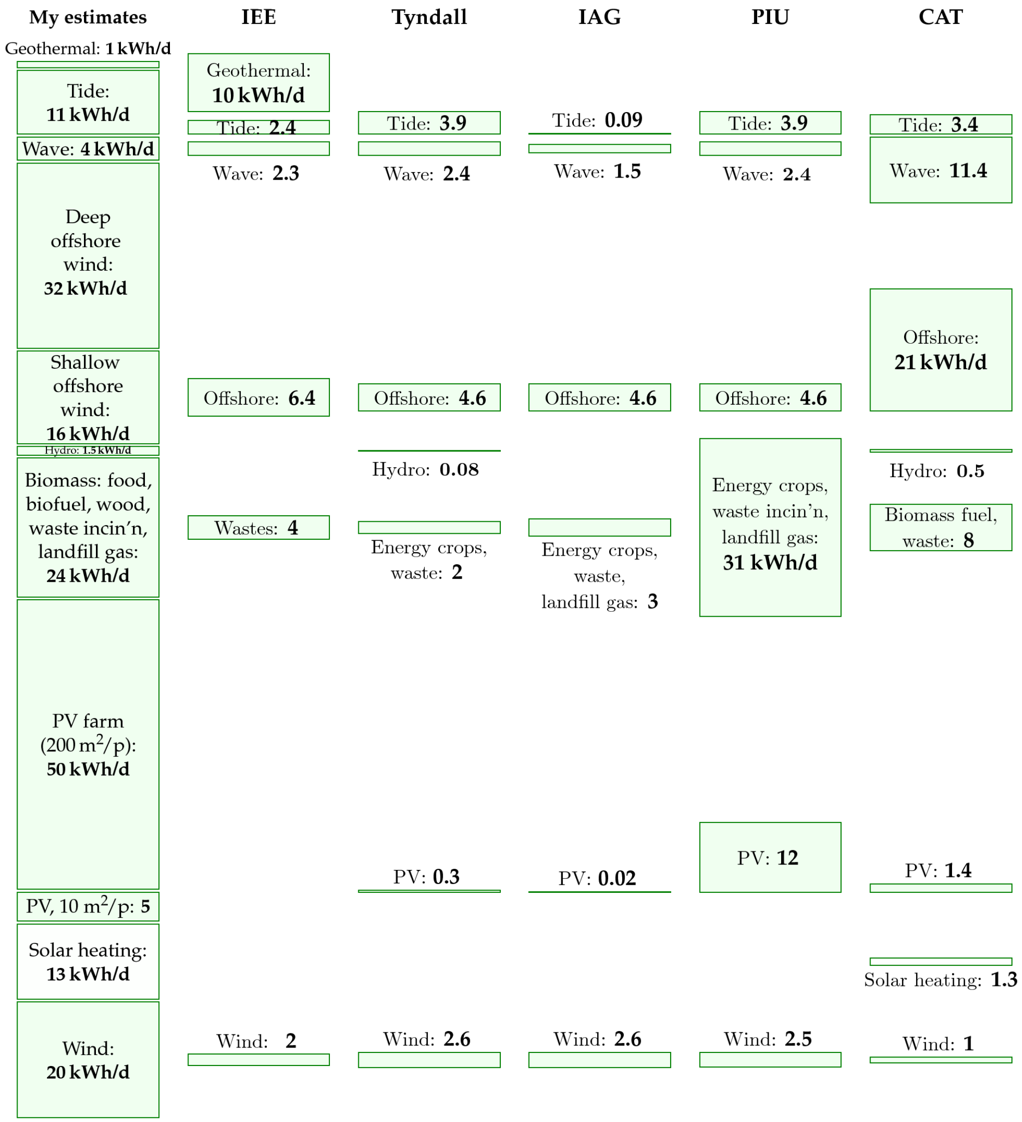
All in all, the red stack *slightly overestimates* current average energy consumption but we were targetting the consumption of a typical affluent person rather than an average person.

Is the UK a good representative of an average European nation?



From this graph we can safely conclude that yes, it is.

Green reflections



Estimates of theoretical or practical renewable resources in the UK by different organisms. ⇒ We have been very optimistic in our estimation of sustainable production.

What would you answer to someone who says:

> We lived on renewables before the industrial revolution. Why not change our lifestyle and go back to the one we had in the middle ages to solve this energy crisis?

Data: In the middle ages, power consumption per day was on average 20 kWh per day per person and everything was biopowered. It required on average 10,000 m^2 of land per person. Available land per person at that time was 52,000 m^2.

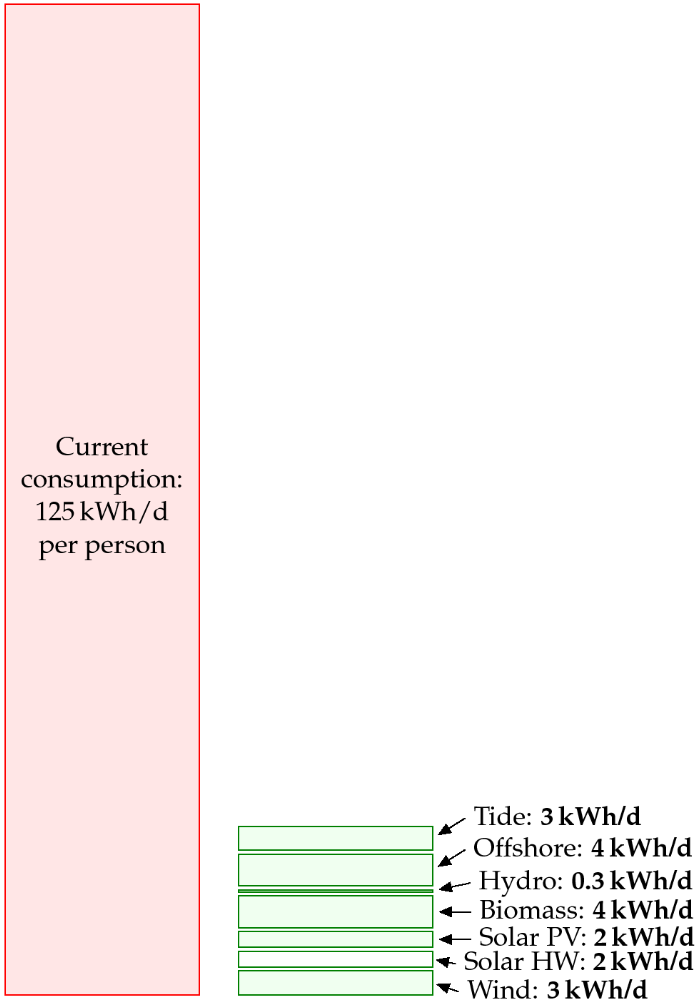
Answer: Even going back to a medevial lifestyle would not solve our energy crisis because the available land per person in the UK is only around 4000 m^2 per person while 10,000 m^2 would be needed.

Green ambitions meet social reality

Typical quotes with respect to renewables:

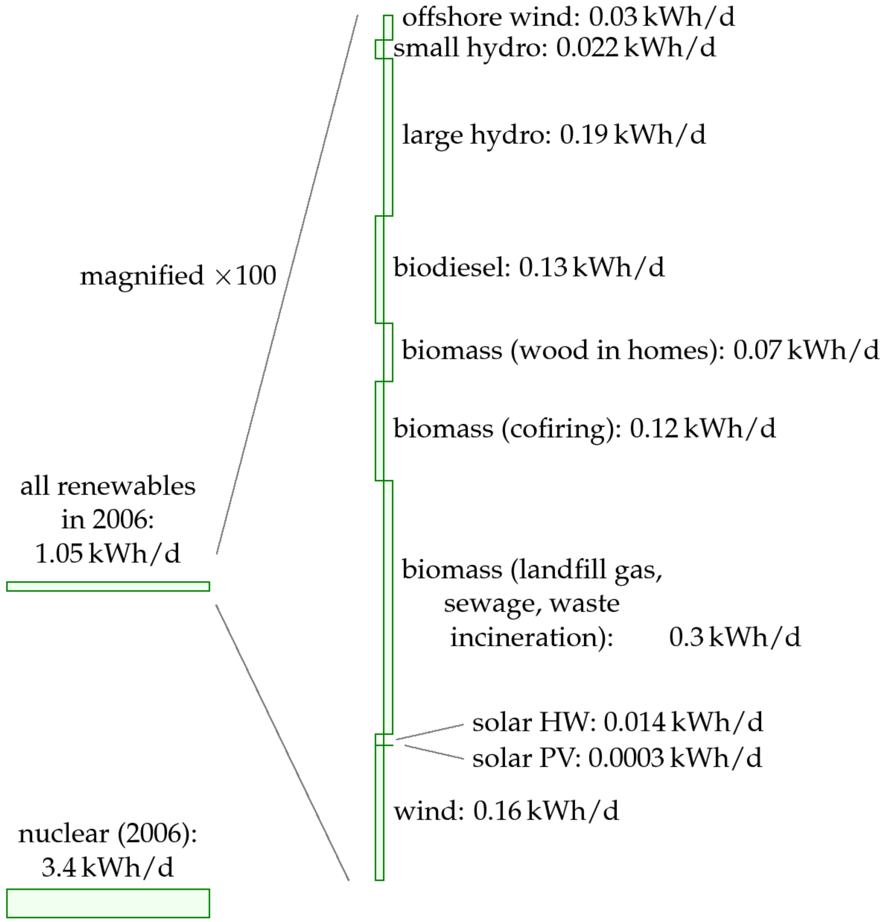
Wind farms? “No they are ugly, noisy and kill bats.”
Solar panels on roofs? “No, they are ugly.”
More biofuels? “No, it ruins the countryside.”
Hydroelectricity? “It ruins the environment and kills fish.”
Offshore wind? “Ugly. It ruins the landscape and I hate these powerlines coming ashore.”
Wave or tidal power “Too much impact on the marine environment.”
Geothermal power “Much too expensive for the power it can produce.”

This is what we may get as renewable energy after public consultation:



There is a possibility that after public consultation, the maximum Britain would get from renewables is around 18 kWh/d per person. This figure is even far less than the primary energy consumption in Britain (125 kWh/d per person).

This is still progress with respect to what we had in 2006:



The breakdown of the renewables on the right-hand side is scaled up 100-fold vertically.

Two main conclusions of Part I

1. To make a difference, renewable facilities have to be country-sized.

Reason behind this fact: renewables are so diffuse.

Power per unit land or water area	
Wind	2 W/m^2
Offshore wind	3 W/m^2
Tidal pools	3 W/m^2
Tidal stream	6 W/m^2
Solar PV panels	5-20 W/m^2
Plants	0.5 W/m^2
Rainwater (highlands)	0.24 W/m^2
Hydroelectric facilities	11 W/m^2
Geothermal	0.017 W/m^2

2. It is not going to be easy to make a plan that adds up using renewables alone.