2. The balance sheet

The question we address in the book: can we conceivably live without fossil fuels?

This will be done through a balance sheet that we will build up progressively:

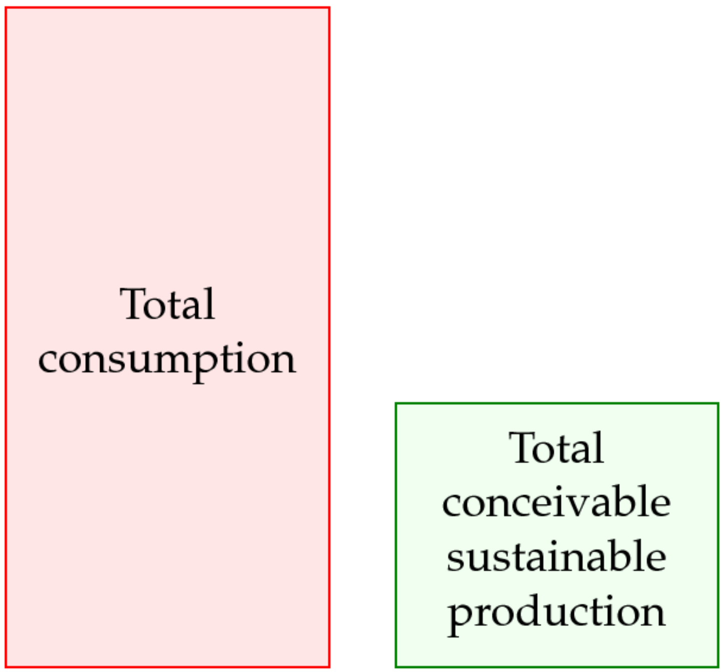
Some key forms of consumption for the left-hand stack will be:

- transport
  - cars, planes, freight
- heating and cooling
- lighting
- information systems and other gadgets
- food
- manufacturing

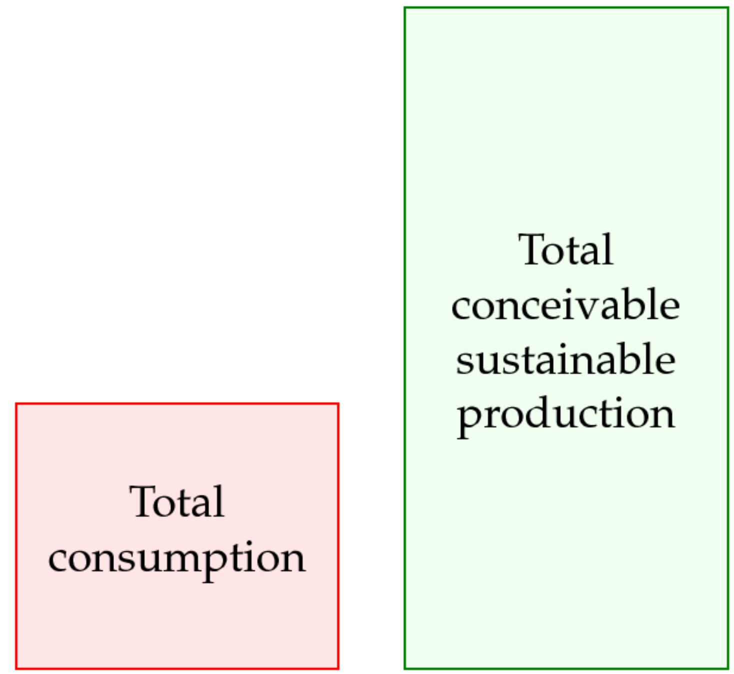
In the right-hand sustainable-production stack, our main categories will be:

- wind
- solar
  - photovoltaics, thermal, biomass
- hydroelectric
- wave
- tide
- geothermal
- nuclear? (with a question-mark, because it's not clear whether nuclear power counts as "sustainable")

The answer will be **NO** if:



The answer will be **YES** if:



Energy and power

Standard units for energy (joule) and power (watt=joule/s) are not convenient here.

Our unit of **energy**: kilowatt-hour (kWh) (1 kWh = 3,6 million joules)

Our unit of **power**: kilowatt-hour (kWh) per day (kWh/d). (40 W $\simeq$ 1 kWh/d)

We also often quote power as **kWh/per day** *per person* so as to better transpose our discussions from one country to another.

The different grades of energy

Energy is always conserved. So, talking of "using" energy does not make a lot of sense. What we really do when using energy is to transform energy from a form that has *low entropy* into a form that has *high entropy*.

We will add labels to the units to distinguish between different grades of energy. One kWh(e) is one kilowatt-hour of electrical energy - the highest grade of energy. One kWh(th) is one kilowatt-hour of thermal energy (the higher the temperature, the lower the entropy). One kWh(ch) is one kilowatt-hour of chemical energy which is also a high-grade energy.

Most of the time, we will talk about energy rather than entropy.

Is it valid to compare different forms of energy such as the chemical energy that is fed into a petrol-powered car with the electricity generated by a wind turbine?

In principle, energy can be converted from one form to another, though conversion entails losses (e.g., fossil fuels used power stations guzzle chemical energy to produce electricity with an efficiency of 40%).

In some summaries of energy production and consumption, different forms of energy are put into the same units but multipliers are introduced (e.g., electrical energy being worth 2.5 times more than the chemical energy in oil).

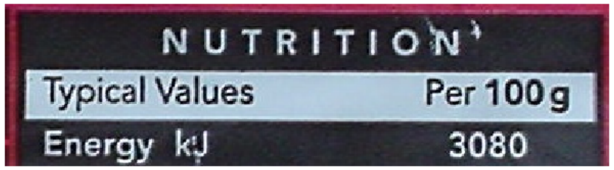
In this class: *one-to-one* conversion rates.

The reason behind this choice: the exchange rate depends on the type of energy that we want. Example: 1kWh of electricity would not be worth 2.5 kWh of chemical energy if we use electricity to make liquid fuels.

3. Cars

$$\text{energy/day} = \frac{\text{distance travelled/day}}{\text{distance/liter of fuel}} \times \text{energy/liter of fuel}$$

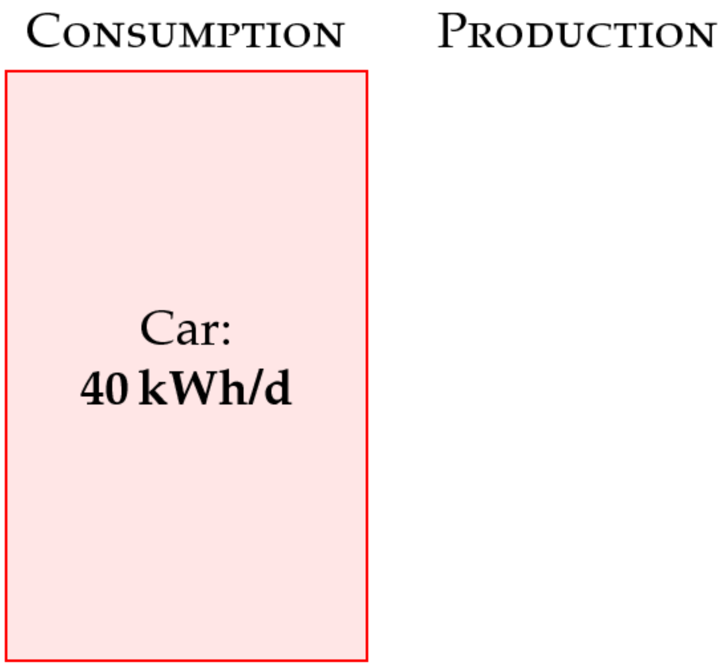
Data/assumptions: (i) distance travelled per day: 50 km (ii) distance per liter of fuel: 12 km (iii) energy per liter of fuel: 10 kWh per liter.



The calorific value of butter, which is also a hydrocarbon, is 3000 kJ per 100 g, or 8 kWh per kg or, assuming a density of 0.8 kg/liter, 7 kWh/liter.

Consumption of a regular car user

$$\text{energy/day} = \frac{50 \text{ km/day}}{12 \text{ km/liter}} \times 10 \text{ kWh/liter} \simeq 40 \text{ kWh/day}$$



Notes:

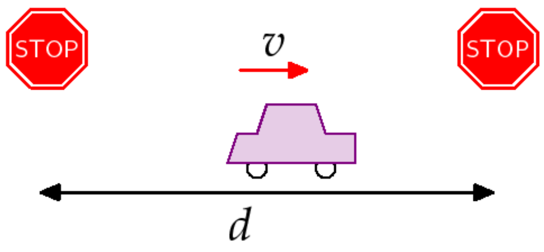
Energy cost of producing the car's fuel. Making one unit of petrol requires an input of 1.4 units of oil.

Energy-cost of manufacturing a car. A full chapter will be dedicated to the "energy for making stuff".

Technical notes on cars

Energy used in cars using fossil-fuels goes to four main destinations: (i) speeding up and slowing down by using the brakes (ii) air resistance (iii) rolling resistance (iv) heat - 75% of the energy is wasted as heat.

First scenario analyzed: rolling resistance neglected; car of mass m_c moves at speed v between steps separated by a distance d.



Questions: How does the energy lost in air resistance compare with the energy lost in the brakes? What is the energy consumption of the car? What can be done to reduce the consumption of the car?

Rate at which energy is transferred to the brakes:

$$\frac{\text{kinetic energy}}{\text{time between braking events}} = \frac{\frac{1}{2}m_c v^2}{\frac{d}{v}} = \frac{\frac{1}{2}m_c v^3}{d}$$

Car creates in a time t a tube of air of volume Avt where A is the area of the front view of the car A_{car} multiplied by a *drag coefficient* c_d. The tube has a mass $m_{air} = \rho Avt$ and swirls at a speed v. It has a kinetic energy equal to $\frac{1}{2}\rho Avtv^2$.

Rate of generation of kinetic energy in swirling air:

$$\frac{\text{kinetic energy tube air}}{t} = \frac{1}{2}\rho Av^3$$

Total rate of energy production by the car =
power going into brakes + power going into swirling air =
$$\frac{1}{2}m_c v^3/d + \frac{1}{2}\rho A v^3$$

• Energy dissipation rate scales as v^3. Energy consumption over a same total distance as v^2.

• Energy lost in air resistance is greater than energy lost in brakes if ratio $(\frac{m_c}{d})/(\rho A)$ is greater than 1 or, equivalently, if $m_c > \rho A d$.

• **Questions:** [A] What is the special distance d^* between stop signs below which the dissipation is braking dominated and above which it is air swirling? [B] What should be done as a function of d to save energy? [C] Can this simple model explain the 40 kWh/d?

[A]

$$d^* = \frac{m_c}{\rho c_d A_{car}} = \frac{1000\ \text{kg}}{1.3\ \text{kg/m}^3 \times \frac{1}{3} \times 3\ \text{m}^2} = 750\ \text{m}$$

[B] If $d < d^*$ (city driving), it is a good idea if you want to save energy:

1. to reduce the mass of the car
2. to get a car with regenerative brakes
3. to drive slower.

If $d > d^*$, energy dissipation is drag-dominated and can be reduced:

1. by reducing the car's drag coefficient
2. by reducing its cross-sectional area; or
3. by driving slower.

[C] Petrol engines are about 25% efficient $\Rightarrow$
total power of the car $\simeq 4[\frac{1}{2}m_c v^3/d + \frac{1}{2}\rho A v^3]$.
Let us assume v = 110 km/h = 31 m/s and $A = c_d A_{car}$ = 1 m^2 and that d is much greater than d^*. Power consumed by the engine:

$$4 \times \frac{1}{2}\rho A v^3 = 2 \times 1.3 \text{ kg/m}^3 \times 1\text{m}^2 \times (31 \text{ m/s})^3 = 80 \text{ kW}.$$

One hour of travel per day $\Rightarrow$ 80 kWh of energy per day. 55 km per day at this speed $\Rightarrow$ 40 kWh.

Comments:

• If you drive the same distance at half the speed, you reduce your consumption by a factor 4 (provided that the engine has the same efficiency, which is not certain).

• *Could a car consume one hundred times less energy on a motorway and still go at 110 km/h?* No, not if it still has the same shape. At best, its fossil-fuel engine could be slightly more efficient.

Rolling resistance

Rolling resistance is caused by the energy consumed in the tyres and bearings of the car, energy that goes into the noise of wheels against asphalt, energy that goes into grinding the rubber off the tyres, and energy that vehicles put into shaking the ground.

Standard model for rolling resistance: a resistance force $F = C_{rr}N$ where C_{rr} is the rolling resistance coefficient and N the force perpendicular to the surface on which the wheel is rolling ($N = m_c g$ if the vehicle is moving on an horizontal plane). A typical value of C_{rr} for a car is 0.01.

Questions: [A] How much power does the engine need to overcome rolling resistance at a speed $v = 110$ km/h $\simeq 31$ m/s? [B] At which speed is a car's rolling resistance equal to air resistance? Data: (i) $A_{car} = 3$ m^2 (ii) m_c=1000 kg (iii) $c_d = \frac{1}{3}$ (iv) ρ= 1.3 kg/m^3 (v) $C_{rr} = 0.01$ (vi) car moving on an horizontal plane.

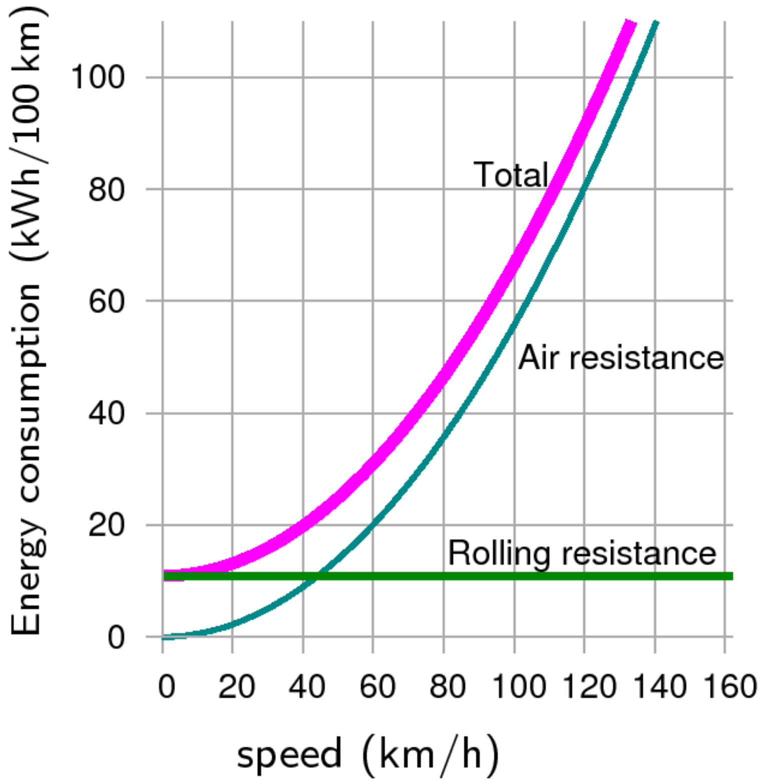
[A] Power required to overcome rolling resistance:

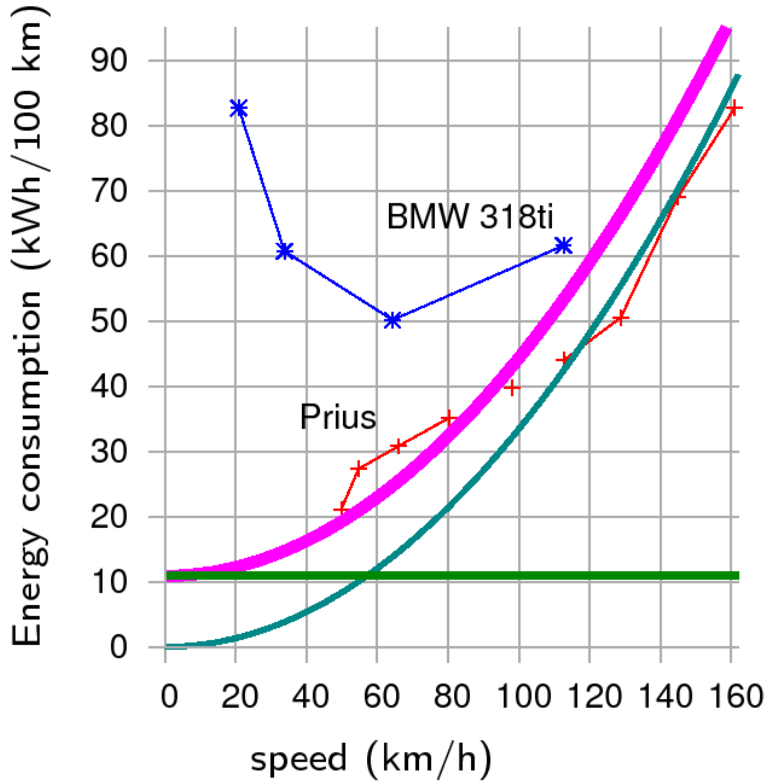
$$\text{force} \times \text{velocity} = 1000 \times 10 \times 0.01 \times 31 = 3100 \text{ W}$$

which, allowing for an engine efficiency of 25% requires 12 kW of power for the engine. Power to overcome drag was 80 kW.

[B] Resistances are equal when : $C_{rr} m_c g = \frac{1}{2}\rho c_d A v^2$, that is when:

$$v = \sqrt{2\frac{C_{rr} m_c g}{\rho c_d A}} \simeq 12.3 \text{ m/s} \simeq 44 \text{ km/h}$$





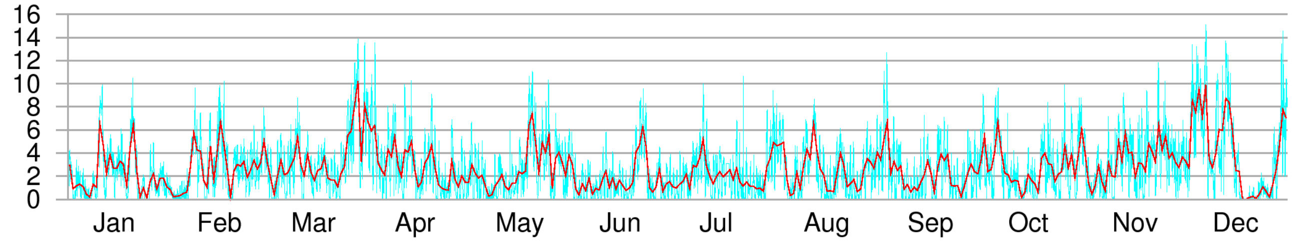
Simple theory of car fuel consumption. Assumptions: energy efficiency 25%; $c_d A_{car} = 1\ \text{m}^2$; $m_{car} = 1000$ kg and $C_{rr} = 0.01$.

Fuel consumption of current cars. This shows that more conservative speed limits will not necessarily lead to energy savings.

4. Wind

How much on-shore wind power could we plausibly generate?

power per person = wind power per unit area × area per person.



Cambridge mean windspeed in m/s, daily (red line) and half-hourly (blue line).

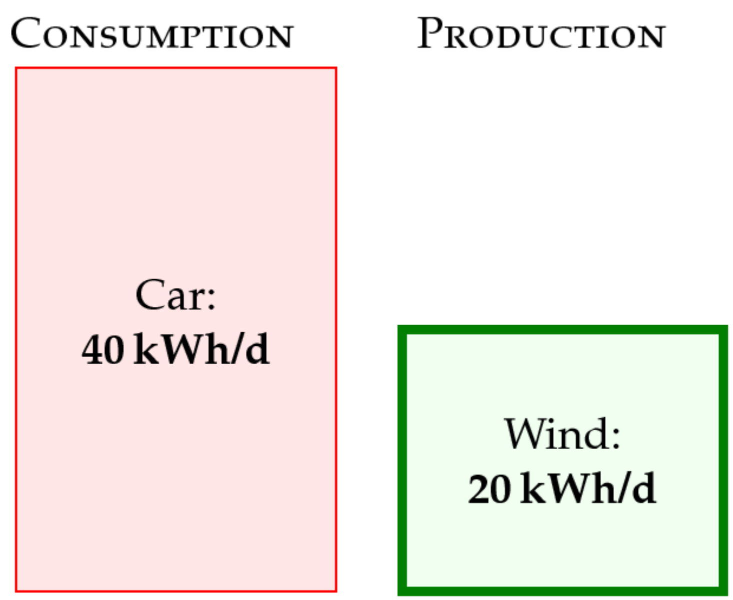
Average wind speed of around 6 m/s ⇒ power per unit area of land 2 W.

Question: What is the maximum amount of wind power (in our favorite unit) that can be generated? Data: 250 people per square kilometer.

Answer:

$2\ \text{W/m}^2 \times 4000\ \text{m}^2\text{/person} = 8000\ \text{W/per person} \simeq 200\ \text{kWh/d per person}$

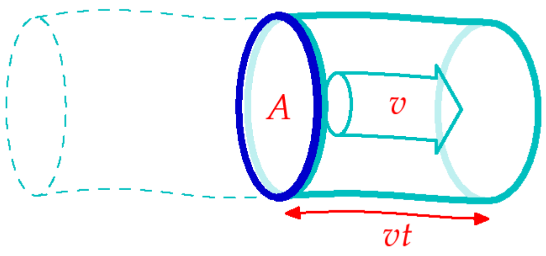
Realistic assumption: only 10% of the country could be covered by windmills $\Rightarrow$ number needs to be reduced to 20 kWh/d per person.



The Whitelee windfarm (near Glasgow):

Description: 140 turbines; covers 55 km^2; *peak* capacity of 322 MW $\Rightarrow$ 6 W/m^2 *peak*. Average amount of power produced is small because turbines do not run at peak output all the time. The ratio of average power to peak power is the load factor. Typical value for a good site with modern turbines: 30% $\Rightarrow$ power production per unit of land for the Whitelee wind farm is $\simeq$ 2 W/m^2.

The physics of wind power



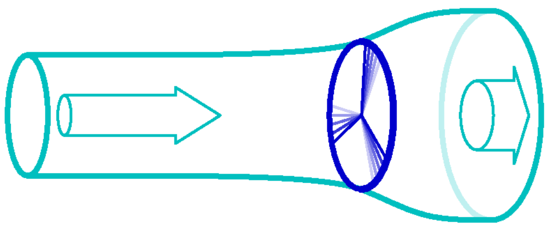
The mass of air that passes through the hoop during a period of time t is equal to $\rho A v t$.

Kinetic energy of that mass of air: $\frac{1}{2}mv^2 = \frac{1}{2}\rho A t v^3 \Rightarrow$ power of the wind for an area A is : $\frac{\frac{1}{2}mv^2}{t} = \frac{1}{2}\rho A v^3$

Question: What is the energy that can be extracted from one square meter of loop? Data: wind speed 6 m/s and density of air 1.3 kg/m^3.

Naive answer: $\frac{1}{2}\rho v^3 = \frac{1}{2} \times 1.3\text{kg/m}^3 \times (6 \text{ m/s})^3 = 140 \text{ W/m}^2$. But a wind mill cannot extract all the kinetic energy of the air otherwise the slowed-down air would get in the way!

The maximum fraction of energy that can be extracted by a disc-like wind mill: $\frac{16}{27} \simeq 0.59$ (result from a German physicist named Albert Beltz). We assume efficiency to be 50% as efficiency to account for other losses.

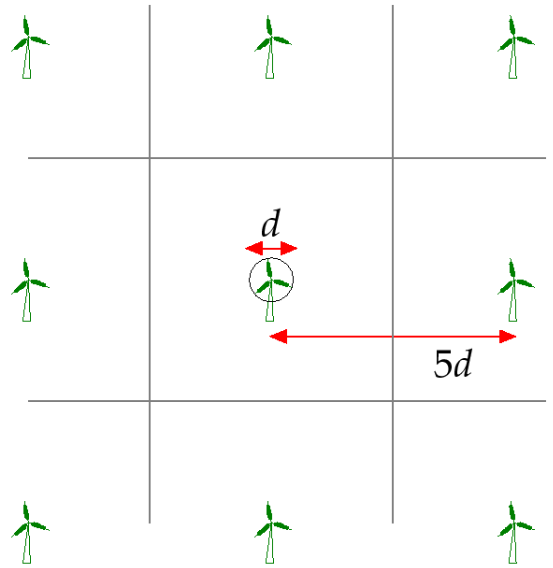


Power of windmill of diameter $d = 25$ m:

$$\begin{aligned} & \text{efficiency factor} \times \text{power per unit area} \times \text{area} \\ = \; & 50\% \times \frac{1}{2}\rho v^3 \times \frac{\pi}{4} d^2 \\ = \; & 50\% \times 140 \text{ W/m}^2 \times \frac{\pi}{4}(25 \text{ m})^2 = 34 \text{ kW} = 816 \text{ kWh/d} \end{aligned}$$

How densely could wind mills be packed?

Problem: too close and those upwind will cast wind-shadows on those downwind. Windmills can't be spaced closer than 5 times their diameter without losing significant power.



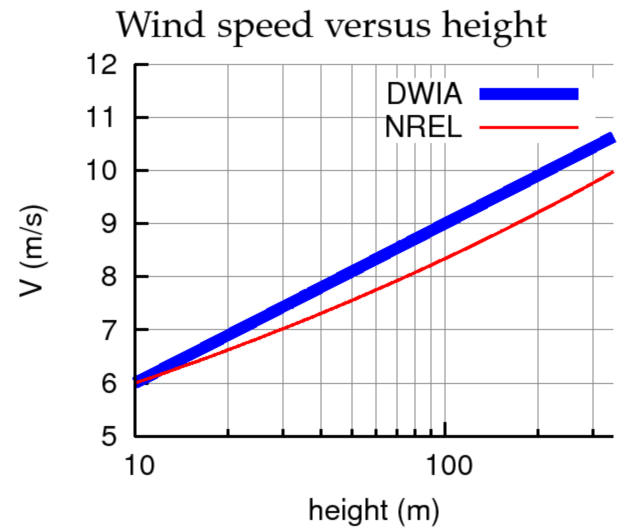
Power that windmills can generate per unit of land:

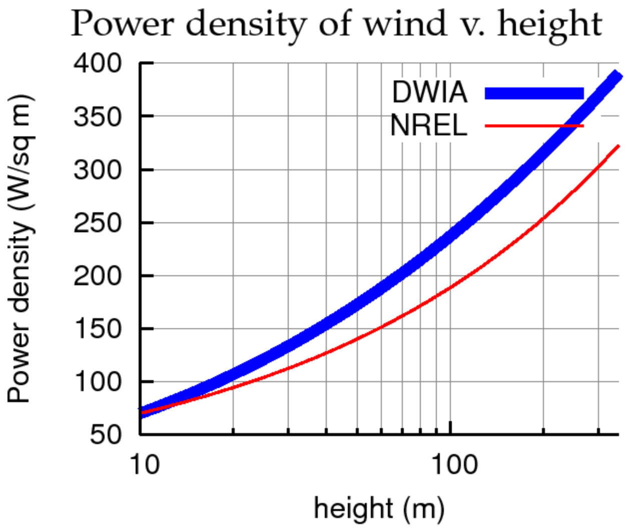
$$\frac{\text{power per wind mill}}{\text{land area per windmill}} = \frac{\frac{1}{2}\rho v^3 \frac{\pi}{8} d^2}{(5d)^2} = 2.2\ \text{W/m}^2$$

Question: Since the answer does not depend on the diameter of the windmill, why are wind mills so big?

Elements of answer: (i) bigger wind mills cost less per MW installed (ii) less land occupation (iii) wind speed increases with height.

1. Wind shear formula from the National Renewable Energy Laboratory (NREL): $v(z) = v_{10}(\frac{z}{10\text{m}})^\alpha$ where v_{10} is the speed at 10 m and α typically in $[0.143, \frac{1}{7}]$.
2. Wind shear formula from the Danish Wind Industry Association (DWIA): $v(z) = v_{ref}\frac{\log(z/z_0)}{\log(z_{ref}/z_0)}$ where z_0 is the roughness length (typical value for agricultural land with houses and hedgerows: 0.1 m) and v_{ref} is the speed at a reference height z_{ref}.





Comments

• In our calculations, we used a mean wind speed of 6 m/s. With a mean wind speed of 4 m/s, we must scale our estimate down, multiplying it by $(4/6)^3 \simeq 0.3$.

• In our calculations, we should not have taken the mean wind speed and cubed it; we should have found the mean cube of the windspeed.

• Installing microturbines on roofs is a bad idea. They usually deliver less than 0.2 kWh per day.

The **Enercon E-126** is the largest wind turbine built to date. Technical specificities: bub height of 135 m; rotor diameter of 126 m; can generate up to 7.58 MW of power (or $\frac{7.58\times10^6\times24}{1000} = 181{,}920$ kWh/d).