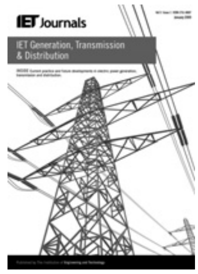
Published in IET Generation, Transmission & Distribution
Received on 28th January 2009
Revised on 1st May 2009
doi: 10.1049/iet-gtd.2009.0046

Special Issue – selected papers on Electricity Markets: Analysis & Operations



ISSN 1751-8687

Design and operation of the locational marginal prices-based electricity markets

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Abstract: Locational marginal prices (LMP)-based electricity markets are implemented in different countries around the world and are dominant in the United States. Even after ten years of the operation, some of the properties of the LMP are not well understood. There are many misconceptions about this clearing mechanism that led to some inefficient market designs. This study is an attempt to consistently present the current state of the LMP-based congestion management, including issues that market and system operators are facing, and analyse new directions of the research. The recommendations are made on which areas are of high priority and should be addressed first. Besides giving a systematic description on how the LMPs are produced, the paper describes both the modelling and implementation challenges and solutions. (This paper solely represents the view point of the author and not necessarily ISO New England's).

Nomenclature

K	number of transmission constraints
k	index of a transmission constraint
N	number of generators in the system
$\boldsymbol{C}$	N-vector of generator offer prices
$\boldsymbol{P}$	N-vector of generator output levels
$\boldsymbol{D}$	vector of nodal loads
$\boldsymbol{e}$	unit vector (all components equal to 1)
Loss	physical system losses
λ	shadow price of the system balance constraint
$\boldsymbol{\mu}$	K-vector of the transmission constraints' shadow prices
i	generator/load index
$\boldsymbol{T}$	$(K \times N)$ matrix of generator shift factors (GSF)
$\boldsymbol{F}^{\max}$	K-vector of transmission limits
$\boldsymbol{P}^{\min}$	N-vector of minimum generator capacity limits
$\boldsymbol{P}^{\max}$	N-vector of maximum generator capacity limits
$\boldsymbol{\eta}^{\min}$	N-vector of the shadow prices of generator lower capacity constraints
$\boldsymbol{\eta}^{\max}$	N-vector of the shadow prices of generator upper capacity constraints
L	Lagrangean
$\boldsymbol{DF}$	vector of all nodal delivery factors
$\boldsymbol{DF}_{\mathrm{m}}$	N_{m}-vector of the marginal unit delivery factors
K_{b}	number of binding transmission constraints
$\boldsymbol{T}_{\mathrm{m}k}$	$(K_{\mathrm{b}} \times N_{\mathrm{m}})$ GSF matrix corresponding only to the marginal unit nodes and binding constraints
$\boldsymbol{C}_{\mathrm{m}}$	N_{m}-vector of the marginal generator offer prices
$\boldsymbol{\Lambda}$	Vector of LMPs for all nodes
$\boldsymbol{\Lambda}_{\mathrm{m}}$	N_{m}-vector of the LMPs at the marginal locations

1 Introduction

Location-based marginal pricing (LBMP) has been one of the most popular means of congestion management in the large number of electricity markets worldwide. It became a part of the standard market design in the United States, and every market in the country either implemented or in the process of implementing LBMP. The idea of using location-based spot pricing of electricity as the congestion management mechanism in electricity markets was proposed in [1, 2]. It was then developed into current locational marginal prices (LMP) framework in the works by Hogan [3] and Hogan *et al.* [4]. In addition to using

LMP, Hogan proposed Financial Transmission Rights (FTRs) as a mechanism to hedge against congestion and preserving physical transmission rights during transition to the market structure. Very rigorous analysis of the locational pricing of electricity was done in [5]. Many issues brought up in these works are still open and are subject of current research.

During more than ten years of running LMP-based markets around the world (New Zealand, Australia, USA etc.), power industry accumulated a wealth of experience and better understanding of the LMP mechanism. Real markets had to deal with practical issues of implementing LBMP and fine-tuning of both the theoretical foundation and practical market design. Some of these issues are: adequacy of the models and tools being used for economic dispatch, unit commitment and LMP calculation; addressing infeasibilities; interpreting LMP components; physical and marginal loss pricing; recovering 'as bid' costs for the generators etc. LMPs have been used not only for pricing energy, but, with the so-called co-optimisation, such ancillary services as reserves and regulation as well. Despite the comparatively large volume of publications dedicated to different topics of location-based spot pricing [6–11], there is still a need for consistent and rigorous description of the current methodology and analysis of different implementations of congestion management systems.

Recently more and more attention is being paid to the design of the clearing auction and pricing of electricity. Both the objective function and the pricing principles are being questioned in recent publications [12–22]. Some of it is caused by the desire of the marketplace to minimise the impact of non-convexities on the market settlements. With only LMPs, a unit may not be able to recover its as-bid cost, including startup and no-load costs. ISOs currently make lump-sum payments to these units for their revenue shortages measured against their as-bid cost. These uplift payments do not send clear price signals to the market, and are often considered as 'out-of-market' payments. The biggest problem with the attempts of pricing unit commitment decisions is that under non-convexity, KKT conditions do not hold true and the market designers have to be extremely careful in defining the pricing mechanism. It is clear that there is no one energy price that can be used to pay for both commitment and dispatch decisions, so some form of non-linear pricing [23] must be utilised. Unlike the convex case, where the pricing is straightforward and the marginal prices are directly derived from the shadow prices, the mixed integer problem solution may not have a clear set of prices that support equilibrium conditions. Recent studies have also suggested two alternative auction objectives to the current bid-cost minimisation. One is to minimise the total consumer payment [16–20]. Depending on the pricing scheme used, the auction formulation could vary significantly and lead to drastically different results. Another alternative auction objective is to maximise the sum of consumer and producer surpluses [21]. The authors argue that the ISO's congestion revenue, which is implicitly included in the current bid-cost objective function, should not be considered as part of the social surplus (SS), and therefore should be excluded from the auction objective. More detailed analysis can be found in [22]. Both action objective and pricing scheme affect energy prices and a very careful analysis is required while designing markets with LMPs.

The paper attempts to describe the current state of the LBMP, its progress during last years and discuss future directions of research. It is structured as follows. Section 2 describes the mathematical model and derivation of the LMPs. Section 3 briefly discusses the use of the optimal power flow (OPF) as the main tool for economic dispatch and LMP calculation. Section 4 describes major properties of LMPs. Next section considers LMP components and the role of the slack bus. Section 6 is dedicated to marginal loss pricing and its effect on market clearing results. Section 7 addresses a very important issue of handling infeasibilities. Finally, Section 8 provides some conclusions and specifies directions of new research.

2 LMP definition and calculation

LMPs are calculated as the result of security constrained economic dispatch (SCED) either in day ahead market (DAM) or real-time market (RTM). The objective of SCED is meeting the load in the power system while maximising social surplus (SS) [24] and honouring operational constraints. It is important to note that SCED must satisfy '$n-1$' criterion that requires no transmission constraints violation under all credible contingencies. In the absence of price-sensitive demand (which is mostly the case in the RTM), maximising SS is equivalent to minimising total production cost or 'as-bid' cost. In the rest of the paper we will use this form of the objective function without losing the generality. The prices are derived from the dual solution of the economic dispatch with commitment statuses of the units fixed. SCED is an OPF program with security transmission constraints and, under the above assumptions, is formulated as follows

$$\operatorname*{Min}_{P} \boldsymbol{C}^{\mathrm{T}}\boldsymbol{P} \tag{1}$$

$$\text{s.t. } \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{P}-\boldsymbol{D}) - \text{Loss} = 0, \quad (\lambda > 0) \tag{2}$$

$$\boldsymbol{T}(\boldsymbol{P}-\boldsymbol{D}) \le \boldsymbol{F}^{\max}, \quad (\boldsymbol{\mu} \le 0) \tag{3}$$

$$\boldsymbol{P}^{\min} \le \boldsymbol{P} \le \boldsymbol{P}^{\max}, \quad (\boldsymbol{\eta}^{\min}, \boldsymbol{\eta}^{\max} > 0) \tag{4}$$

where shadow prices are shown in parenthesis next to each corresponding constraint. Loss, being a non-linear function of $\boldsymbol{P}$, is usually replaced by its linear approximation. Different approaches are being used in different markets

IET Gener. Transm. Distrib., 2010, Vol. 4, Iss. 2, pp. 315–323
doi: 10.1049/iet-gtd.2009.0046

(see Section 3). The LMP is defined as a change in production cost to optimally deliver an increment of load at the location, while satisfying all the constraints. From this definition, at the optimal point, taking into account complementarity conditions [25], LMP at bus i, λ_i, can be obtained as the partial derivative of the Lagrangean of (1)–(4)

$$L = \boldsymbol{C}^{\mathrm{T}}\boldsymbol{P} - \lambda(\boldsymbol{e}^{\mathrm{T}}(\boldsymbol{P}-\boldsymbol{D}) - \mathrm{Loss}) - \boldsymbol{\mu}^{\mathrm{T}}(\boldsymbol{T}(\boldsymbol{P}-\boldsymbol{D}) - F^{\max}) + \boldsymbol{\eta}^{\max}(\boldsymbol{P}-\boldsymbol{P}^{\max}) + \boldsymbol{\eta}^{\min}(-\boldsymbol{P}+\boldsymbol{P}^{\min}) \quad (5)$$

$$\lambda_i = \frac{\partial L}{\partial D_i} \quad (6)$$

Owing to the convexity of the linear problem, the KKT conditions hold true. After substituting (5) into (6) and taking into account that $(\partial \mathrm{Loss}/\partial D_i) = -(\partial \mathrm{Loss}/\partial P_i)$, we obtain

$$\lambda_i = \lambda - \frac{\partial \mathrm{Loss}}{\partial P_i}\lambda + \sum_{1}^{K} T_{ik}\mu_k \quad (7)$$

This is the way LMPs are calculated in most of the implementations. Alternatively, this can be done by including base case (pre-contingency) power flow equations as additional constraints into optimisation problem. In this case, LMPs at each bus are just the shadow prices of the nodal balance equations. This model is often referred to as DC OPF-based economic dispatch. This will be discussed in more details in the next section.

Currently, there are two major approaches to calculate LMPs in RTM: ex post and ex ante. In the market environment, the spot electricity prices should provide sufficient incentives for all the dispatchable resources to follow dispatch instructions. Both ex ante and ex post prices combined with proper pricing rules can achieve this goal. NY ISO uses ex ante prices as the real-time prices and penalises non-performing resources on the basis of reduced generation quantity [26], whereas ISO NE, PJM and MISO adopt the ex post pricing that provides dispatch incentives on the ground of rational prices [27, 28]. There are two major methodologies for ex post energy pricing. One starts from the dual of the ex ante dispatch problem [3, 5] and formulates the ex post pricing as a linear programming problem to find a set of prices that can reflect the actual behaviour of generating resources. However, this method assumes that resources' actual performance is optimal, which is almost never true in real-time operations. In addition, the objectives of the pricing problem (maximise or minimise the congestion revenue) is not well defined or justified. Another approach, which has been adopted by PJM and ISO NE [29], uses an incremental optimisation of the ex ante dispatch problem according to resources' actual operating conditions. At the same time, pricing rules are also established to ensure that the ex post pricing takes generating resources' performance into consideration. In practice, this works well, although it uses some empirical formulae. Both ex ante and ex post approaches have their own benefits and problems. For example, ex ante pricing does not have a capability to penalise non-performing units (as required by the majority of market rules), whereas ex post pricing can implement this. Ex post pricing has significant difficulties in implementing co-optimisation of the energy and reserves. In general, there is no rigorous foundation and justification for using one or another. This requires further research by economists and power engineers.

3 Linear against non-linear OPF

As mentioned above, SCED is being used both in DAM and RTM. In the vast majority of implementations, this is an LP-based OPF [30–32]. It utilises successive LP to find a solution of the non-linear OPF problem. From the practical perspective, LP-based implementation is significantly more robust and faster than non-linear programming method. Successive LP makes use of the decomposition between optimisation and contingency analysis (CA) by generating only violated post-contingent constraints and feeding them into optimisation procedure. This allows one to significantly improve the solution speed and, assuming the convexity of the original problem, solve AC OPF. Despite (1)–(3) being a linear problem, together with CA, it produces non-linear optimisation solution. This fact is usually overlooked, and current SCED is often considered being a DC OPF. The DC OPF, in contrast with LP-based or successive OPF, uses DC power flow in CA as well and solves linear approximation of the SCED. Voltage constraints are usually not modelled in the SCED; instead, the units that are required for voltage support are prescheduled ensuring very small deviation of the voltages from the scheduled profile. This proved to be quite an efficient practical way of working with the linear model. Another problem of modelling reactive power/voltage relations in SCED is dealing with the quality of the model and reactive power management. Both for the DAM and RTM, the forecast and metering of reactive power is not as robust and accurate as active power. Most of the state estimators are very well tuned for the active power, but deficient in reactive power estimation.

While feeding post-contingent GSF into optimisation procedure, practical implementations usually apply filtering to eliminate very small shift factors. This allows avoiding 'unusual' and very inefficient from the operating practices perspective dispatch patterns. Such situations usually occur when most of the effective controls are exhausted and the software attempts to move very ineffective units to control the congested constraint.

The full AC formulation of the problem is very large and complex to be used in production today. One of the attempts to solve full non-linear problem with security constraints is the so-called super-OPF [33]. This is the research conducted at Cornell University in an attempt to preserve full non-linear structure of the problem and identify the scale of the differences with other simplified methods. In this case, one has to solve a problem with tremendous number of constraints explicitly including post-contingent power flow equations for each contingency as additional constraints. With today's model sizes and thousands of contingencies, the dimension can reach very high numbers and additional research will be required to meet performance requirements. Another issue is ensuring the convexity of the problem. Non-linear AC-based formulation in case of non-convexity creates a pricing problem, because of KKT conditions not being met. If not using 'super-OPF' formulation, linearisation of at least security constraints is inevitable.

4 Major properties of LMPs

If LMPs are calculated as the result of solving linear programming problem, there are several important properties that can be observed. There will always be a subset of units that are marginal; the rest of the units will be either at their minimum or maximum output level. The LMP at each marginal unit location will always be equal to its offer price. This property directly follows from the KKT conditions

$$\frac{\partial L}{\partial P_i} = 0 = C_i - \lambda\left(1 - \frac{\partial \text{Loss}}{\partial P_i}\right) - \sum_1^K T_{ik}\mu_k + \eta^{\max} - \eta^{\min}$$
$$\Rightarrow \lambda_i = C_i + \eta^{\max} - \eta^{\min} \tag{8}$$

For the marginal units, both $\eta^{\max}$ and $\eta^{\min}$ equal zero, and $\lambda_i = C_i$. This is true for systems with congestion and losses. From (8), it is also easy to see the relations between offer prices and LMPs for all infra-marginal units. Every unit dispatched at its maximum output would have its LMP higher than the offer price ($\lambda_i = C_i + \eta^{\max}$), and the unit dispatched at its minimum output would have the LMP lower than the offer price ($\lambda_i = C_i - \eta^{\min}$).

In non-degenerate case, if there are n binding constraints, there are $(n+1)$ marginal units. All the LMPs in the system are determined by the marginal units' offer prices; moreover, each LMP is a linear combination of these offer prices. Indeed, if the economic dispatch problem is linear and has a unique solution, all active (binding) constraints, including unit capacity constraints, turn into equalities. At the optimal point

$$\begin{cases} \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{P} - \boldsymbol{D}) - \text{Loss} = 0 \\ P_i = P_i^{\min} \text{ or } P_i^{\min},\ i = 1, \ldots, N_{nm} \\ \sum_1^n T_{ik}(P_i - D_i) - F^{\max} = 0,\ k = 1, \ldots, K_{\mathrm{b}} \end{cases} \tag{9}$$

In order to have a unique solution of (9), the following must hold: $N_{\mathrm{m}} = N - N_{\mathrm{nm}} = K_{\mathrm{b}} + 1$.

In order to derive LMPs as a function of the marginal offer prices C_{m}, we can use (7). LMPs at marginal locations

$$\Lambda_{\mathrm{m}} = \boldsymbol{C}_{\mathrm{m}} = \boldsymbol{DF}_{\mathrm{m}} \cdot \lambda + \boldsymbol{T}_{\mathrm{m}\boldsymbol{k}} \cdot \boldsymbol{\mu} \tag{10}$$

This is a system of $(k+1)$ linear equations with $(k+1)$ variables: λ and $\boldsymbol{\mu}$. Solving (10), we obtain the shadow prices

$$\begin{bmatrix} \lambda \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} (\boldsymbol{DF}_{\mathrm{m}})^{\mathrm{T}} \\ \boldsymbol{T}_{\mathrm{m}\boldsymbol{k}} \end{bmatrix}^{-1} \cdot \boldsymbol{C}_{\mathrm{m}} \tag{11}$$

Now all LMPs can be expressed as the linear function of the marginal offers

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{DF} \\ \boldsymbol{T}_{\boldsymbol{k}} \end{bmatrix} \cdot \begin{bmatrix} (\boldsymbol{DF}_{\mathrm{m}})^{\mathrm{T}} \\ \boldsymbol{T}_{\mathrm{m}\boldsymbol{k}} \end{bmatrix}^{-1} \cdot \boldsymbol{C}_{\mathrm{m}} \tag{12}$$

Note that (12) does not contain shadow prices. This equation can be used to efficiently forecast LMPs. Predicting very few binding constraints and marginal units, one can calculate prices at all locations. This is much easier than trying to forecast the LMP at every location. ISO has all the information needed to obtain very accurate forecast. Expression (12) can also be used to evaluate the impact of different marginal units on the LMP at a specific location by estimating the contribution of the particular unit to the LMP. More general sensitivity analysis is provided in [34].

5 LMP components and the role of the slack bus

Currently, because of the need to price FTRs, LMPs are being split into three components

$$\begin{aligned} &\lambda && -\text{'Energy' component} \\ &\lambda^{\mathrm{L}} = -\frac{\partial \text{Loss}}{\partial P_i}\lambda && -\text{Loss component} \\ &\lambda^{\mathrm{C}} = \sum_i^K T_{ik}u_k && -\text{Congestion component} \end{aligned} \tag{13}$$

Looking at (7), it is obvious that $\lambda_i = \lambda + \lambda^{\mathrm{L}} + \lambda^{\mathrm{C}}$. The names for the first two components are misleading. The energy component is very often thought of as the price of energy if there were no losses and congestion. This is not correct: the 'energy' component is actually a price of energy at the slack bus [35] location. In fact, it will change with the change of the slack location with the dispatch staying the same. The loss component more accurately should be called marginal loss component and it does not reflect the cost of physical losses. We will discuss this in the next section.

doi: 10.1049/iet-gtd.2009.0046

Loss sensitivities (loss factors – LF) and power flow sensitivities (GSF) are used in calculating LMP components out of dual variables – shadow prices. By definition, both LF and GSF require a selection of the slack bus and are dependent on its location. In other words, depending on the selection of the slack bus, the same LMP can be decomposed into different sets of components. This issue becomes very important when marginal loss pricing is introduced in the market clearing. In the system without losses, there is no need to split LMP into components because of the fact that energy components are the same for every location. The difference in LMPs between two locations equals the difference in congestion components. With the introduction of losses in the model, it is not true anymore, and this is the only reason that LMP-based markets today split LMPs into components. Moreover, FTRs do not provide the hedge against the difference in LMPs anymore, only for the differences in congestion components. It would be more appropriate to split LMPs into two components by combining 'energy' and loss components into one component that could be called 'delivered energy component'

$$\lambda_i^{\mathrm{DE}} = \left(1 - \frac{\partial \mathrm{Loss}}{\partial P_i}\right)\lambda = \mathrm{DF}_i \cdot \lambda \qquad (14)$$

where DF_i is a traditional delivery factor [35]. Another important conclusion is that each LMP component by itself does not have any meaning and should not be used in any market rules, settlements, or statistical analysis.

While designing market with LMPs, it is important to make sure that at least the LMP values are not changing with the change of the slack bus, otherwise it would be almost impossible to come up with participant-neutral location. As it is shown in [36], under traditional model, moving the slack changes the optimisation problem and would produce different LMPs for different slack locations. The Energy Management System (EMS) software also can automatically change the location of the slack bus depending on the topology and current state of the system. The effect of the slack change can be significantly reduced by switching from a single slack to the distributed slack as it was proposed in [36]. The paper also shows that it can be done only for the market clearing process by converting sensitivities produced by the EMS to a distributed slack – market reference. Additional measures must be taken to assure full LMP independence of the reference choice [36].

Another fact that is somewhat counterintuitive is that the LMPs in the system can exceed the highest offer price. This fact was ignored in many market designs, where the caps on the offers were introduced in anticipation that this would keep prices limited to this cap. In reality, the price can reach almost arbitrarily high value that is much higher than the highest bid [37, 38]. That can be easily shown by using (12).

The necessity to use linear representation of the losses requires careful modelling decisions. For example, in New Zealand [39], the implementation of the SCED includes DC power flow model and losses are linearised for each individual branch by using piece-wise linear approximation. Because the DC power flow model does not have losses by definition, the losses for each line are added to the receiving end bus. The alternative methodology was proposed in [36] and is being used in ISO New England, MISO and PJM. This method linearises system losses with load-weighted distribution to all load buses in the system. It is shown in the paper that not only LMPs are independent on the selection of the slack bus, but congestion components as well. That property is very important to preserve the value of the FTRs and congestion revenue fund when the market reference changes. In [40], the authors attempt to derive 'slack bus independent' LMP components, but also make some arbitrary assumptions, allowing several location to have zero loss components and making different locations non-comparable. Very good analysis summarising different approaches to LMP components calculation is done in [9]. The authors admit that because of non-linearity of the problem, there is no 'exact' solution to the problem and the decision on how to define components is not technical, but a policy one. The best resolution would be not using components at all, which will require additional research in the area of congestion hedging and surplus allocation.

6 Marginal loss pricing

Another issue that has been controversial in LMP market design is marginal loss pricing. The importance of marginal loss pricing, especially in the markets with large and very large footprints (PJM and MISO, for example) is obvious, so most of the LMP markets either implemented or about to implement marginal loss modelling. The major misunderstanding is usually in treating loss components of the LMP as payment for losses, which is wrong. The price of one MWh of physical losses in the LMP-based market is undefined. It is impossible to assign single price to physical losses under LMP mechanism.

Let us consider the energy settlement from the ISO perspective. All loads pay the LMP at their location – this is a credit, EC, for the ISO (positive)

$$\mathrm{EC} = \sum_{i=1}^{N_d} \lambda_i D_i \qquad (15)$$

The ISO pays the generators the LMP at their locations – debit, ED, for the ISO (negative)

$$\mathrm{EC} = -\sum_{i=1}^{N_d} \lambda_i P_i \qquad (16)$$

Overall energy revenue collection, ER, by the ISO is the following

$$\mathrm{ER}=\sum_{i=1}^{N_d}\lambda_i D_i-\sum_{i=1}^{N_d}\lambda_i P_i \tag{17}$$

Substituting (14) into (17)

$$\mathrm{ER}=\sum_{i=1}^{N_d}\left(\lambda-\frac{\partial \mathrm{Loss}}{\partial P_i}\lambda+\sum_{k=1}^{K}T_{ik}\mu_k\right)D_i - \sum_{i=1}^{N_d}\left(\lambda-\frac{\partial \mathrm{Loss}}{\partial P_i}\lambda+\sum_{k=1}^{K}T_{ik}\mu_k\right)P_i \tag{18}$$

After regrouping terms, we have

$$\mathrm{ER}=-\lambda\sum_{i=1}^{N}(P_i-D_i)+\lambda\sum_{1}^{N}\frac{\partial \mathrm{Loss}}{\partial P_i}(P_i-D_i) - \sum_{1}^{K}\mu_k T_{ik}(P_i-D_i) \tag{19}$$

Analysing (19), it is easy to see that $\sum_{i=1}^{N}(P_i-D_i)=\mathrm{Loss}\cdot\sum_{1}^{K}T_{ik}(P_i-D_i)=F^{\max}$ for the binding constraints and 0 for non-binding constraints (because of corresponding $\mu_k=0$). Then we can rewrite (19) as follows

$$\mathrm{ER}=-\lambda\sum_{i=1}^{N}\mathrm{Loss}+\lambda\sum_{1}^{N}\frac{\partial \mathrm{Loss}}{\partial P_i}(P_i-D_i)-\sum_{1}^{K}\mu_k F^{\max} \tag{20}$$

Now we can see that only the first term contains payments for physical losses. As in the uniform price market, this is the debit because of the imbalance between generation and load. It can only be suggested that the price of MWh of physical losses equals the energy price at the market reference. There is no exact theory under this statement, and any alternative could be considered, but this seems to be the most logical assumption. At the same time, it is important to understand that defined in this way the price of physical losses will be dependent on the location of the market reference. In this case, payments for physical losses are being offset by the loss revenue fund collected by the ISO. No LMP market in the USA has explicit design for charging loads for losses directly. The third term is the congestion revenue fund that is used to pay FTR holders. Taking into account the assumptions used in the formulation (1)–(4), transmission constraints' shadow prices, μ_k, are negative, so the congestion fund is always positive. Separate components should not be interpreted as payments for energy, losses and congestion, respectively – LMP is the price of energy at a location.

Both loss revenue and congestion revenue funds are dependent on the location of the market reference – moving slack bus will change the distribution of money between congestion and loss funds. The nature of both funds is actually the same – surplus of funds collected by the ISO is inherent to LMP. Unfortunately, the current market designs are dealing with them completely differently. The congestion fund is distributed to the FTR holders. FTRs are auctioned by the ISO and the objective of the auction is to maximise the benefits of the bidders. Besides 'physical' bidders that own assets and have positions in the energy market, financial players can also buy FTRs to claim part of the congestion fund. At the same time, loss revenue fund is wrongfully considered to be an 'over-collection' of money for physical losses. The fund is allocated as such to the loads. Different allocators are used in different markets [41–44], but all of them are based on the wrong assumption of the loss fund's nature. This fund, as much as the congestion fund, does not belong to anybody specifically. There have been attempts to implement 'Loss FTRs' [45, 46], but without much success. Most of the time, it is very difficult to achieve revenue adequacy with loss FTRs. Today the issue of the loss revenue allocation is not a technical problem, but rather a policy issue. This requires further economic analysis and research. In the non-linear system, these two funds are non-separable and any split is conditional.

7 Handling infeasibilities

One of the major challenges in market design and implementation is reconciling market clearing with operational practice. Lack of information in economic dispatch software may cause software to produce results that do not satisfy system operator. Operators' experience and knowledge of the situation in the system that is not formalised into information fed into the software requires sometimes their intervention. It is desirable to design and implement software that minimises the need for the operator's discretion, especially when it affects the prices. Solving this problem is still more an art than a science and requires more rigorous economic and engineering foundation.

Some of the markets implement 'look ahead' capabilities in their real-time economic dispatch trying to mimic operator's trending capability. This is especially used in order to commit fast start resources to meet forecasted load changes 2–4 h ahead. NYISO uses multi-interval optimisation simulating 2-h-ahead unit commitment and dispatch [26]. While improving the dispatch, look-ahead approach still cannot eliminate infeasibilities.

One of the most common infeasibilities is the transmission constraint violation. This happens because of the lack of controls available to the SCED. These can be generators ramp rates, regulation requirements, capacity constraints etc. In this situation, either some units that are electrically very far away from the constrained area may be dispatched, causing very large shadow prices and high LMPs, or penalty price creates extremely wide price spreads, including very high negative prices. Quite often this

doi: 10.1049/iet-gtd.2009.0046

constrained situation may last only for one or two dispatch intervals whereas, for example, additional unit is ramping up. Most of these constraints are thermal $(n-1)$ constraints, which have comparatively low probability of occurrence. Under the above conditions, it is acceptable to relax the limits by most of the time just several MWs.

Another approach is relaxing the penalty – set it to a certain cap. This method can control the price spreads and preserve correct incentives to follow dispatch instructions. The disadvantage of this approach is the difficulty in determining and justifying the value of the cap and setting the priority of multiple penalties if multiple violations occur at the same time. Some of the ISOs are adapting this approach by applying a threshold to the transmission constraint shadow prices.

One more example of the infeasibility that is very difficult to resolve is excess generation condition, when total online generation is greater than the system load. All dispatchable units are dispatched at their economic minimum, which could actually be higher than their physical minimum. The major challenge from the market prospective is producing a set of rational prices. The prices should incent loads to consume more and the generators to lower their output. ISO New England, for example, declares the event and sets all prices to zero and is planning in the nearest future to introduce negative bid pricing (already in use in the MISO). The problem still requires additional research and economic solution. Different markets use different approaches under excess generation conditions.

Very different from the above, infeasibility stems from the 'preventive' nature of today's economic dispatch that produces control actions that should satisfy $(n-1)$ criterion. One of the requirements is to make sure that the post-contingent flow should be brought back to normal [47] rate level within pre-specified time interval after contingency. This requirement is not usually reflected in the SCED formulation and the results of dispatch cannot guarantee that the generators will have enough ramping capability to lower the flow within the required time. System operators have to very closely monitor these conditions and sometimes have to interfere by, for example, artificially lowering thermal limits to 'pre-ramp' the units. To handle this kind of infeasibility, the method similar to the one proposed in [48] can be used. The resolution would be adding additional constraints on corrective actions to the economic dispatch formulation, requiring sufficient ramping capability after contingency. This will require using Benders decomposition (or another alternative approach) to solve the optimisation problem; hence very efficient implementation will be required. This is a subject for a separate discussion.

8 Conclusions

As LMP market design continues to be widely utilised in different markets, more research is needed in economic, mathematical and engineering foundations of the methodology and efficient implementation approaches. Although the current methodology is comparatively robust, the issues described in the paper require more scrutiny in order to make congestion management and market auctions more efficient. The following main topics require attention and should be given high priority in research and development:

1. The use of AC-based OPF.

2. Rigorous justification and calculation of ex post and ex ante pricing; justification of the prices based on multi-period optimisation in economic dispatch.

3. Marginal loss pricing and hedging mechanisms against the differences in LMPs and congestion components. Loss revenue allocation mechanism.

4. Rationalisation of the prices while dealing with infeasibilities.

5. Pricing integer decisions both in unit commitment and economic dispatch.

9 Acknowledgment

The author would like to thank Dr F. Zhao and Dr T. Zheng for fruitful discussions on the subject of this paper. This paper solely represents the viewpoint of the author and not necessarily ISO New England's.

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