Chapter 4. Participating in Markets for Electrical Energy

Previously: we have discussed the basic principles of electricity markets.

Now: we discuss the decisions that generators, consumers and others take to optimize their benefits.

Which others? Storage facilities, hybrid participants.

Market not perfectly competitive $\Rightarrow$ optimization needs to be done while taking into account the behavior of other participants.

The consumer's perspective

If they pay a flat rate for electricity $\Rightarrow$ demand only affected by the cycle of their activities. Averaged over a few months, their demand reflects their willingness to pay this flat rate.

What if the price fluctuates more rapidly? Almost no demand response because price elasticity for the demand is usually small.

Value Of Loss Load (VOLL): is the estimated amount that customers receiving electricity with firm contracts would be willing to pay to avoid a disruption in their electricity service.

	Minimum	Maximum	Average
January 2001	0.00	168.49	21.58
February 2001	10.00	58.84	18.96
March 2001	8.00	96.99	20.00

Pool selling price: electricity pool of England and Wales (in £/MWh). VOLL for the same period 2768 £/MWh.

Small elasticity partially explained by the fact that VOLL much greater than average price of electricity.

Shifting demand

Shifting demand: rather than reducing their demand, the consumers may decide to delay this demand until the prices are lower. This concept exists for a long time with for example the night and day tariffs.

There exist many opportunities for shifting demand that can still be exploited, even for small customers (e.g., turning off the fridge for half an hour, delaying a laundry).

Investments in systems to exploit these shifting demand opportunities important in a landscape where more and more electricity is produced by renewables.

Investments: recording consumers' consumption for every market period (essential for not purchasing anymore electricity on the basis of a tariff), automatic devices installed in homes for shifting loads, etc.

Retailers for electrical energy

Small consumers usually prefer purchasing on a tariff = constant price per kilowatt-hour rather than to be active participants. They buy their energy to a retailer.

Challenge: to buy energy at a variable price on the wholesale market and sell it a fixed price at the retail level.

The quantity-weighted average price at which a retailer purchases energy should be lower than the rate it charges its customers.

Must forecast very well the consumption of its consumers to reduce its exposure to spot market prices (accuracy usually good if a large group of customers).

Retailers may offer more competitive tariffs to customers which record the energy consumed at every time period.

Example

Period	Units	1	2	3	4	5	6	7	8	9	10	11	12	Average	Total
Load forecast	(MWh)	221	219	254	318	358	370	390	410	382	345	305	256	325	3828
Contract purchases	(MWh)	221	219	254	318	358	370	390	410	382	345	305	256	325	3828
Average costs	($/MWh)	24.70	24.5	27.50	35.20	40.70	42.40	45.50	48.60	44.20	38.80	33.40	27.70	36.10	
Contract costs	($)	5459	5366	6985	11 194	14 571	15688	17745	19 926	16884	13386	10187	7091	12 040	144482
Actual loads	(MWh)	203	203	287	328	361	401	415	407	397	381	331	240	330	3954
Imbalances	(MWh)	−18	−16	33	10	3	31	25	−3	15	36	26	−16	10.5	
Spot prices	($/MWh)	13.20	12.50	17.40	33.30	69.70	75.40	70.10	102.30	81.40	63.70	46.90	18.30	50.35	
Balancing costs	($)	−238	−200	574	333	209	2337	1753	−307	1221	2293	1219	−293	742	8901
Total costs	($)	5221	5166	7559	11 527	14 780	18025	19498	19 619	18 105	15679	11406	6798	12 782	153 383
Total revenues	($)	7815.5	7815.5	11 050	12628	13 899	15439	15 978	15 670	15285	14669	12744	9240	12686	152 229
Profits	($)	2595	2650	3491	1101	−882	−2587	−3521	−3950	−2821	−1011	1338	2442	−96	−1154
Profits w/o error	($)	3050	3066	2794	1049	−788	−1443	−2730	−4141	−2177	−104	1556	2765	241	2896

Flat retail price: 38.50 $/MWh.

The producer's perspective

We focus on a generating company that tries to maximize the profits it derives from a single generating unit called unit i.

Problem: $\max \Omega_i = \max[\pi.P_i - C_i(P_i)]$ where P_i= power produced and $C_i(P_i)$ cost of producing P_i.

Optimality when $\frac{d\Omega_i}{dP_i} = \frac{d(\pi.P_i)}{dP_i} - \frac{dC_i(P_i)}{dP_i} = 0$.

First term = marginal revenue of unit i (MR_i). Second term = marginal cost of production of unit i (MC_i).

For optimality: $MR_i = MC_i$.

Basic dispatch

Competition is supposed to be perfect. Price π not affected by P_i.

Optimality when $\frac{dC_i(P_i)}{dP_i} = \pi$.

As long as π is given, scheduling of the units can be done independently.

If P_i solution greater than P_i^{max} generator $\Rightarrow P_i = P_i^{\text{max}}$. If P_i less than P_i^{min}, additional check needs to be done to be sure that the generator will not be loosing money.

Example

Consider the unit with the inverse production function (quantity of fuel needed to generate P_1) $H_1(P_1) = 110 + 8.2P_1 + 0.002P_1^2$ MJ/h with a minimum stable generation is 100 MW and a maximum output of 500 MW.

Cost of fuel $F = 1.3$ \$/MJ.

Questions: (I) What is the power that should be generated by the unit to maximize profit if electricity can be sold at 12 \$/MWh? (II) At which electricity prices should the unit operate at maximum output? (III) What is the electricity price below which the unit cannot make any profit?

(I)

Cost of production if F (in \$/MJ) is cost in fuel per \$/MJ:
$110F + 8.2P_1F + 0.002P_1^2F$ \$/h.

Since $F = 1.3$ \$/MJ $\Rightarrow C_1(P_1) = 143 + 10.66P_1 + 0.0026P_1^2$ \$/h.

Optimality condition: $\frac{dC_1(P_1)}{dP_1} = 10.66 + 0.0052P_1 = 12$ \$/MWh $\Rightarrow$
$P_1 = 257.7$ MW.

Solution valid because in between P_{min} and P_{max}.

(II)

$\frac{dC_i(P_i{=}500)}{dP_i} = price \Rightarrow price = 13.26$ \$/MWh.

(III)

Can be computed by solving the following minimization problem:

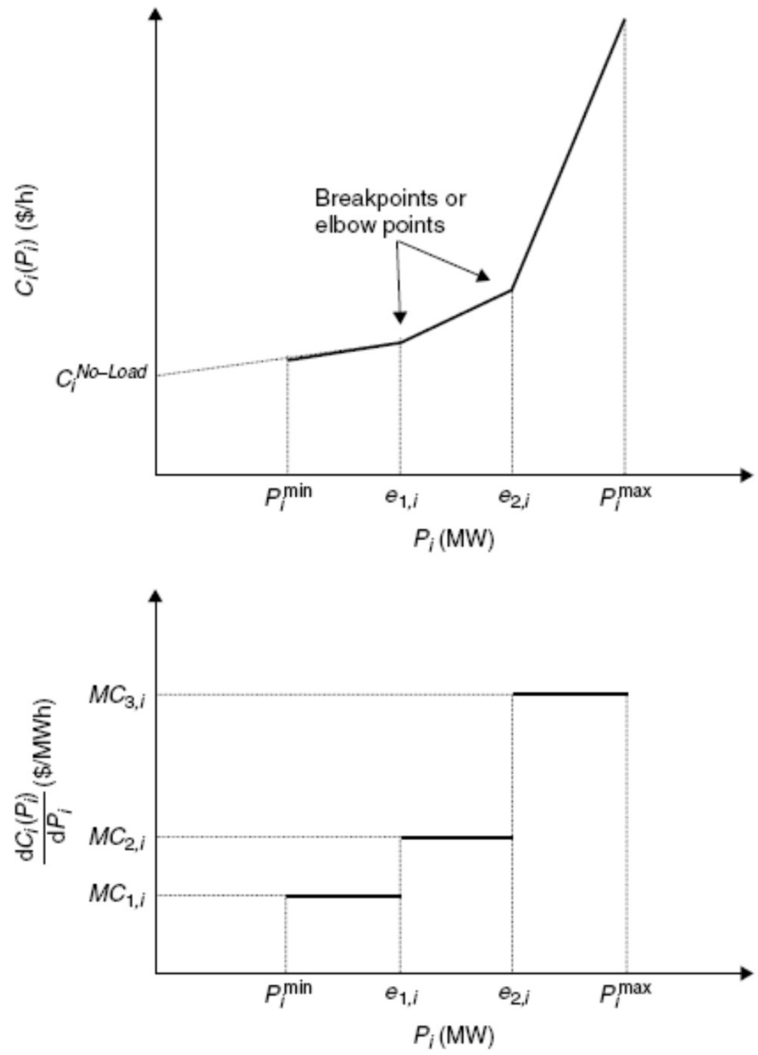
$$\min_{price, P_1} price$$

under the following contraints:

$$price \times P_1 - H_1(P_1)F \geq 0$$

$$P_1 \geq 100$$

$$P_1 \leq 500$$



Question: What is the optimal dispatch for a market price π when having these piecewise linear cost curves?

More realistic scheduling

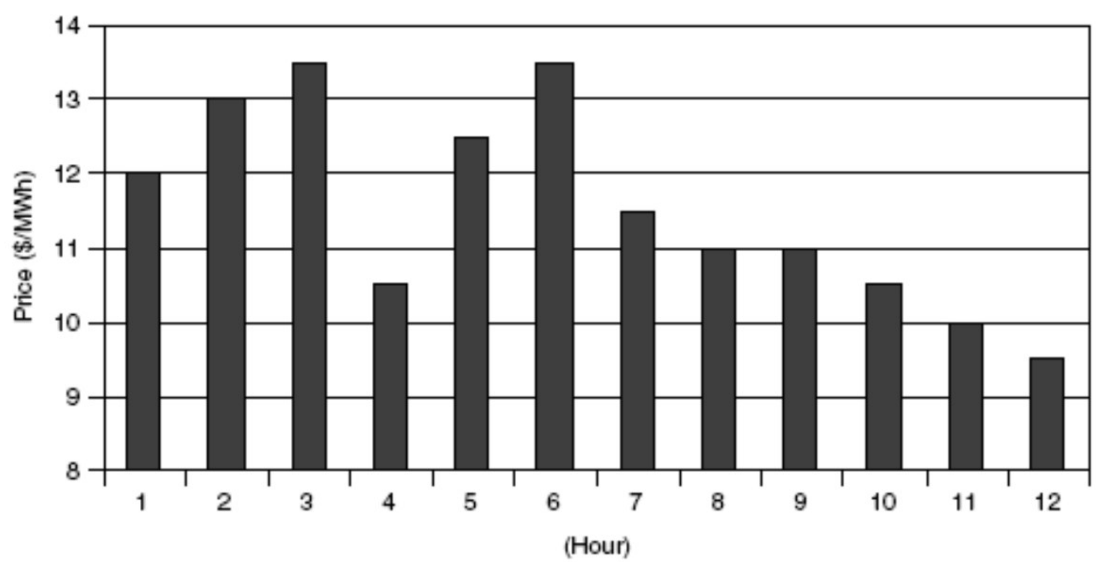
The production profile needs to be optimized over several market periods rather than one due to (among others):

Start-up costs: costs of starting units. Diesel generators and open cycle gas turbines = low start-up costs. Large thermal units: large amount of heat energy before the steam is at a temperature and pressure that are sufficient to sustain the generation of electric power. They have large start-up costs.

Dynamic constraints: Limits placed on the variation of production of a generator to avoid mechanical stress (mainly of the prime mover) and all the problems related to gradients in temperature.

Environmental constraints: E.g.: rate at which a certain pollutant is released in the atmosphere is limited (or total over one year); constraints on the use of water for hydro plants.

Hour	1	2	3	4	5	6	7
Price ($/MWh)	12.0	13.0	13.5	10.5	12.5	13.5	11.5
Generation (MW)	257.7	450.0	500.0	100.0	353.8	500.0	161.5
Revenue ($)	3092	5850	6750	1050	4423	6750	1858
Running cost ($)	3063	5467	6123	1235	4240	6123	1933
Start-up cost ($)	600	0	0	0	0	0	0
Total cost ($)	3663	5467	6123	1235	4240	6123	1933
Profit ($)	–571	383	627	–185	183	627	–75
Cumulative profit ($)	–571	–188	439	254	437	1064	989



The production versus purchase decision

Suppose that a generation company has signed a contract for the supply of a given load L during a single hour. How should it use its portfolio of N generating plants?

Problem: Minimize $\sum_{i=1}^{N} C_i(P_i)$ subject to $\sum_{i=1}^{N} P_i = L$ where P_i represents the production of unit i of the portfolio and $C_i(P_i)$ the cost of producing this amount of power with this unit.

Solution: form the Lagrangian function l, compute the values of the variable that sets its partial derivatives equal to zero to get necessary conditions for optimality.

$l(P_1, P_2, \ldots, P_N, \lambda) = \sum_{i=1}^{N} C_i(P_i) + \lambda(L - \sum_{i=1}^{N} P_i)$ where λ is the Lagrangian multiplier.

These conditions can be written:

$$\frac{\partial l}{\partial P_i} = \frac{dC_i}{dP_i} - \lambda = 0 \qquad \forall i = 1, \ldots, N$$

$$\frac{\partial l}{\partial \lambda} = (L - \sum_{i=1}^{N} P_i) = 0$$

From there: $\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \ldots = \frac{dC_N}{dP_N} = \lambda$.

Lagrange multiplier is thus equal to the cost of producing one additional megawatt-hour with any of the generating units $\Rightarrow$ often called the shadow price of electricity.

If market price π lower than shadow price λ, the company should buy electricity on the market (decrease L) up to the point at which: $\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2} = \ldots = \frac{dC_N}{dP_N} = \pi$

Imperfect competition

It is quite common for an electricity market to consist of a few strategic players and a number of price takers.

The few strategic players may play on the fact that they influence the price. Often they own several units. The price π is no longer a variable on which the firm cannot act.

The total profit of a firm f that owns multiple generating units is $\Omega_f = \pi.P_f - C_f(P_f)$ where:
(i) P_f = total output
(ii) $C_f(P_f)$ = minimum cost for producing P_f.

Ω_f does not depend anymore here only on P_f !

The Nash equilibrium

Let $\Omega_f = \Omega_f(X_f, X_{-f})$ where X_f represents the actions (called also the strategic variables) of firm f and X_{-f} those of its competitors.

If other firms behave in a rationale way, it is "reasonable" for each firm f to select the strategic variable X_f^* such that:
$\Omega_f(X_f^*, X_{-f}^*) \geq \Omega_f(X_f, X_{-f}^*) \quad \forall X_f \forall f$
where X_{-f}^* represents the optimal action of the other firms.

The strategic profile (X_f^*, X_{-f}^*) is the Nash equilibrium of a noncooperative game.

A model of strategic interaction is required for computing a Nash equilibrium.

The Cournot model

Cournot model: a model of strategic interaction where the quantities P_f are the strategic variables.

What are the elements required for computing a cournot equilibrium?
[1] The strategic space of each firm (set of values for P_f)
[2] The production cost function $C_f(P_f)$ of each firm
[3] The inverse demand curve $\pi(P)$

If n firms and if each strategic space has m elements $\Rightarrow$ the condition $\Omega_f(X^*_f, X^*_{-f}) \geq \Omega_f(X_f, X^*_{-f}) \quad \forall X_f \forall f$ needs to checked for m^n strategic profiles to find all Nash equilibria.

Example of computation of a Cournot equilibrium

We consider the case of two firms (A and B) that compete for the supply of electricity.

[1] Strategic space for each firm: $\{5,10,15,20,25,30,35\}$ MW
[2] $C_A = 35.P_A$ \$/h, $C_B = 35.P_B$ \$/h
[3] Inverse demand function $\pi = 100 - (P_A + P_B)$ \$/h

Demand	Profit of A
Profit of B	Price

Production of firm B \ Production of firm A	5		10		15		20		25		30		35	
5	10	275	15	500	20	675	25	800	30	875	35	900	40	875
	225	90	200	85	175	80	150	75	125	70	100	65	75	60
10	15	250	20	450	25	600	30	700	35	750	40	750	45	700
	400	85	350	80	300	75	250	70	200	65	150	60	100	55
15	20	225	25	400	30	525	35	600	**40**	**625**	45	600	50	525
	525	80	450	75	375	70	300	65	**225**	**60**	150	55	75	50
20	25	200	30	350	35	450	40	500	45	500	50	450	55	350
	600	75	500	70	400	65	300	60	200	55	100	50	0	45
25	30	175	35	300	40	375	45	400	50	375	55	300	60	175
	625	70	500	65	375	60	250	55	125	50	0	45	−125	40
30	35	150	40	250	45	300	50	300	55	250	60	150	65	0
	600	65	450	60	300	55	150	50	0	45	−150	40	−300	35
35	40	125	45	200	50	225	55	200	60	125	65	0	70	−175
	525	60	350	55	175	50	0	45	−175	40	−350	35	−525	30

A more analytical approach

Profits firm A: $\Omega_A(P_A, P_B) = \pi(P_A + P_B).P_A - C_A(P_A)$
Profits firm B: $\Omega_B(P_A, P_B) = \pi(P_A + P_B).P_B - C_B(P_B)$

For each of these problems we can write a condition of optimality:
$\frac{\partial \Omega_A}{\partial P_A} = \pi(P_A + P_B) - \frac{dC_A(P_A)}{dP_A} + P_A . \frac{d\pi}{dP} . \frac{dP}{dP_A} = 0$
$\frac{\partial \Omega_A}{\partial P_B} = \pi(P_A + P_B) - \frac{dC_B(P_B)}{dP_B} + P_B . \frac{d\pi}{dP} \frac{dP}{dP_B} = 0.$
where $P = P_A + P_B$.

This gives the reaction curves: $P_A = \frac{1}{2}(65 - P_B)$ and $P_B = \frac{1}{2}(55 - P_A)$. Solving these equations gives the same equilibrium as before: $P_A = 25$ MWh, P_B= 15 MWh and $\pi = 60$ $/MWh.

Plants with very low marginal costs

Several types of plants (nuclear, hydroelectric, renewable) have (almost) negligible marginal costs $\Rightarrow$ challenge is to recover the investments.

Nuclear units: must operate at an almost constant generation level. Owners must sell the nominal power of their units at every hour and almost at any price.

Hydro plants (with substantial reservoir): can adjust their production; must forecast the periods when the price for electricity will be the highest and sell during these periods.

Wind farms and solar plants: production uncontrollable; require accurate prediction techniques to be able to sell their production at (not too) unfavorable prices.

The Hybrid Participant's Perspective

Hybrid Participant: behaves like producers and consumers depending on the circumstances.

Pumped hydro plan most common type of hybrid participant.

Operation profitable if the revenue of selling energy during periods of high prices is larger than the cost of the energy consumed during periods of low prices.

Calculation needs to take into account losses ! Around only 75% of the energy consumed can be sold back to the market for a pumped hydro plant.

Homework

For a group of students, explain what a load aggregator is. Present a company which is acting as load aggregator and its business model.

For a group students, present the concept of smart meter and the impact of smart meters on the electrical sector as a whole.

For a group of students, present a methodology published in a recent research paper (less than 5 years old) for forecasting loads.

For a group of students, present a methodology published in a recent research paper (less than 5 years old) for forecasting spot prices.

The presentations should not last more than 20 minutes.