Chapter 5. Transmission networks and electricity markets

Introduction

In most of the regions of the world: assumptions that electrical energy can be traded as if all generators were connected to the same busbar not tenable.

Transmission constraints and losses can introduce gross distortions in the market for electrical energy.

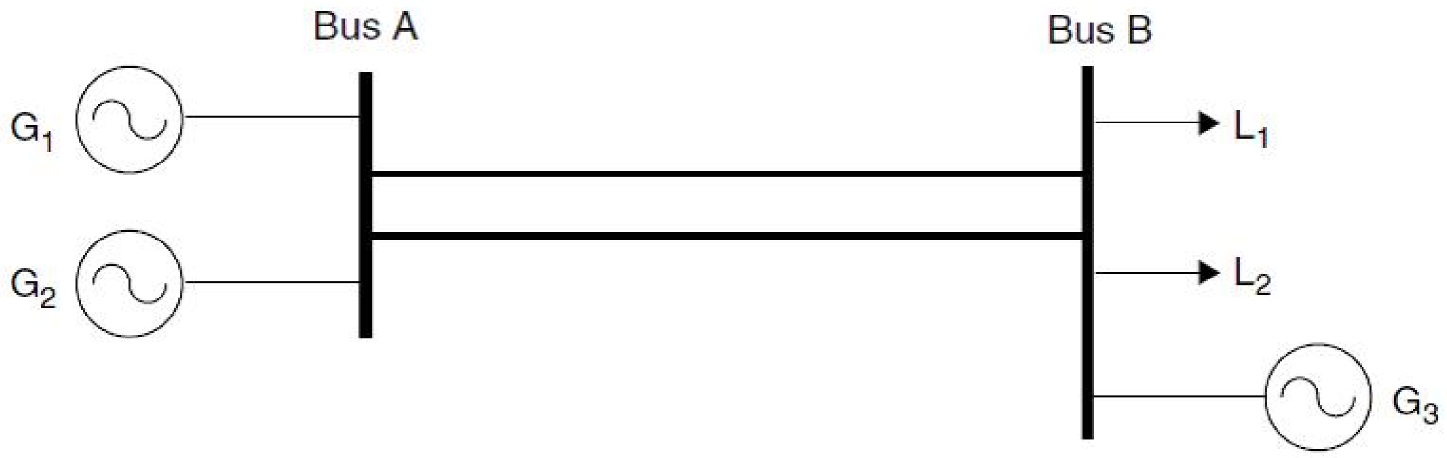
In this lesson: we study the effects that a transmission network has on trading of electrical energy and the special techniques that can be used to hedge against these limitations.

Decentralized trading over a transmission network

In a decentralized or bilateral trading, transactions for electrical energy involve only two parties: a buyer and a seller.

System operator not involved in these transactions and does not set the prices at which the transactions take place. Role limited to:
1. Buying or selling energy to balance the load and the generation.
2. Limiting the amount of power that generators can inject at some nodes of the system if security cannot be maintained by other means.

Bilateral trading in a two-bus power system: example



Data: L_1 =300 MW and $L_2 =$ 200 MW.

If transmission lines between Bus A and Bus B are always able to transfer 500 MW, even under contingency conditions, than the transactions never need to be curtailed. But otherwise, the SO may have to intervene.

How to curtail transactions?

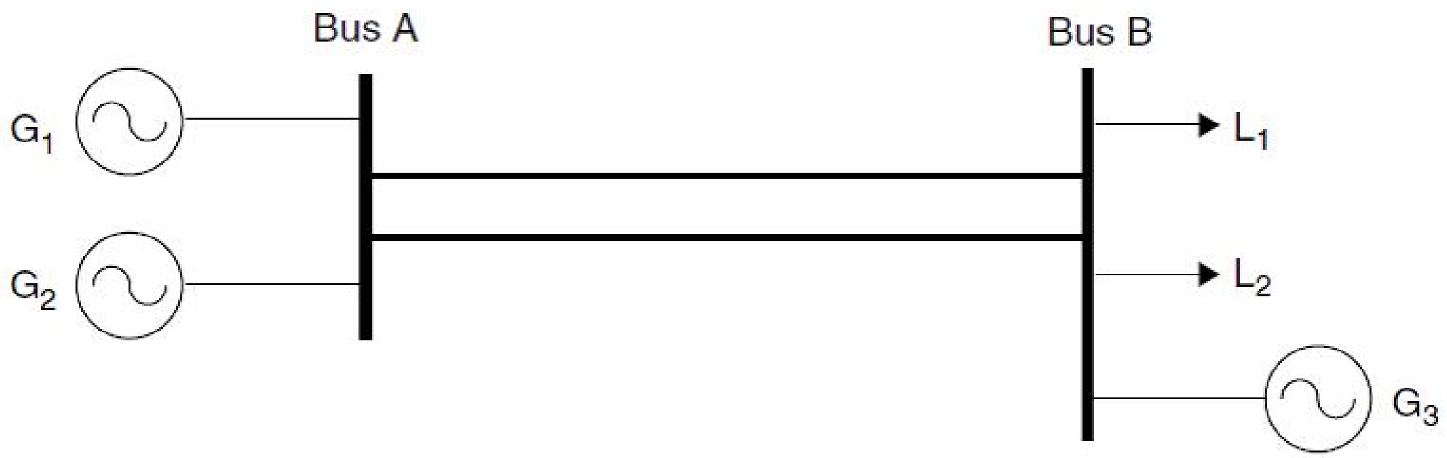
Determining whether a set of transactions would make the operation of the system insecure is relatively easy, even if computationally demanding.

But what about determining which transactions should be curtailed?
1. Administrative procedures can be established to determine the order in which transactions should be cut back (based on the nature of the transaction, their order of registration or historical data)
2. These administrative curtailments are however inefficient and should be avoided because they do not factor in the relative economic benefits of the various transactions that are unknown to the SO in a decentralized trading system.

Physical transmission rights

Advocates for decentralized trading believe that buyers and sellers are best placed to decide whether they wish to use the network. When they sign a contract, buyers and sellers shoud therefore be offered the possibility to purchase the right to use the transmission system for this transaction.

Physical transmission rights purchased at auctions. Parties have the opportunity to decide whether these additional costs are justifiable.



Suppose G_1 and L_1 (300 MW) have agreed on a price of 30$/MWh while G_2 and L_2 (200 MW) on a price of 32$/MWh. At the same time, G_3 offers energy at 35$/MWh.

$\Rightarrow$ L_2 should not pay more than 3$/MWh for the transmission rights. L_1 could pay up to 5$/MWh before buying energy from G_3.

Problems with transmission rights

Two main problems with transmission rights:

1. The path that power takes through a network not determined by the wishes of market participants but by physical laws. Even if it was determined by the wishes of market participants, issues would still pertain due to the fact that power can be traded from A to B and B to A.

2. Physical transmission rights can exacerbate market power.

Electricity not transmitted by trucks

Let us assume two parallel paths (A and B) connecting node 1 to node 2. The impedances of these paths are z_A and z_B, respectively. Let $\overline{I}$ be the current flowing from A to B.

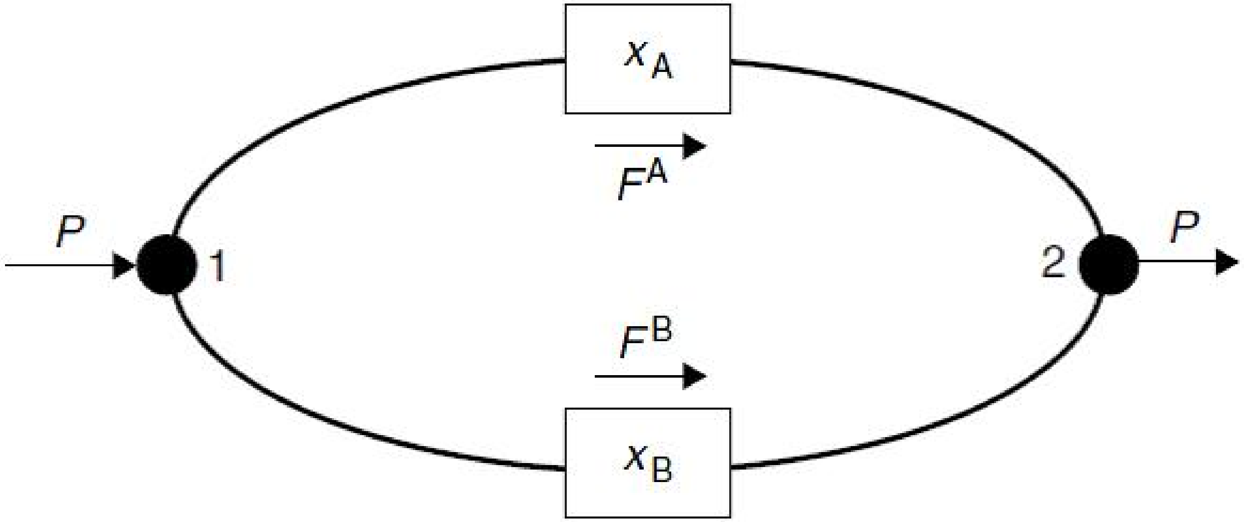
Voltage difference between nodes 1 and 2: $\overline{V_{12}} = z_A \overline{I_A} = z_B \overline{I_B}$. Since $\overline{I} = \overline{I}_A + \overline{I}_B$ we have:

$$
\begin{aligned}
I_A &= \frac{z_B}{z_A + z_B} \overline{I} \\
I_B &= \frac{z_A}{z_A + z_B} \overline{I}
\end{aligned}
$$

To simplify discussion, we assume that the resistance in any branch is much smaller that its reactance: $Z = R + jX \simeq jX$.

Let P be the active power flowing from A to B ($P = Re(\overline{V} \times \overline{I}^*)$). Let F^A be the active power going through A and F^B the active power going through B. We have:

$$F^A = \frac{z_B}{z_A + z_B} P$$
$$F^B = \frac{z_A}{z_A + z_B} P$$

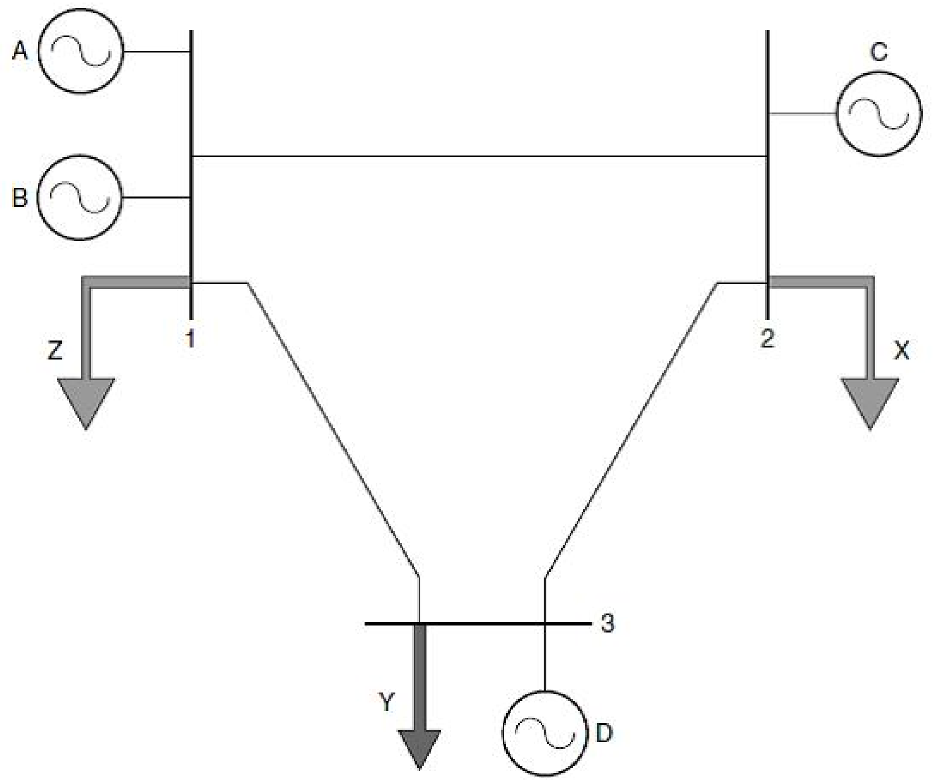


Factors relating the power injections to the branch flow are called the power transfer distribution factors (PTDF).

Why does it make sense to set limit on lines in terms of MW when trading electricity?

Limits should naturally be set as an upper bound on the number of amperes that can be safely carried through the line and not in terms of maximum active power.

However, if (i) the reactive power that flows through a line is close to zero and (ii) the voltage remains constant, then we have $P = Constant \times |\bar{I}|$ and setting a limit on P makes sense.



Branch	React. (p.u.)	Cap. (MW)
1-2	0.2	126
1-3	0.2	250
2-3	0.1	130

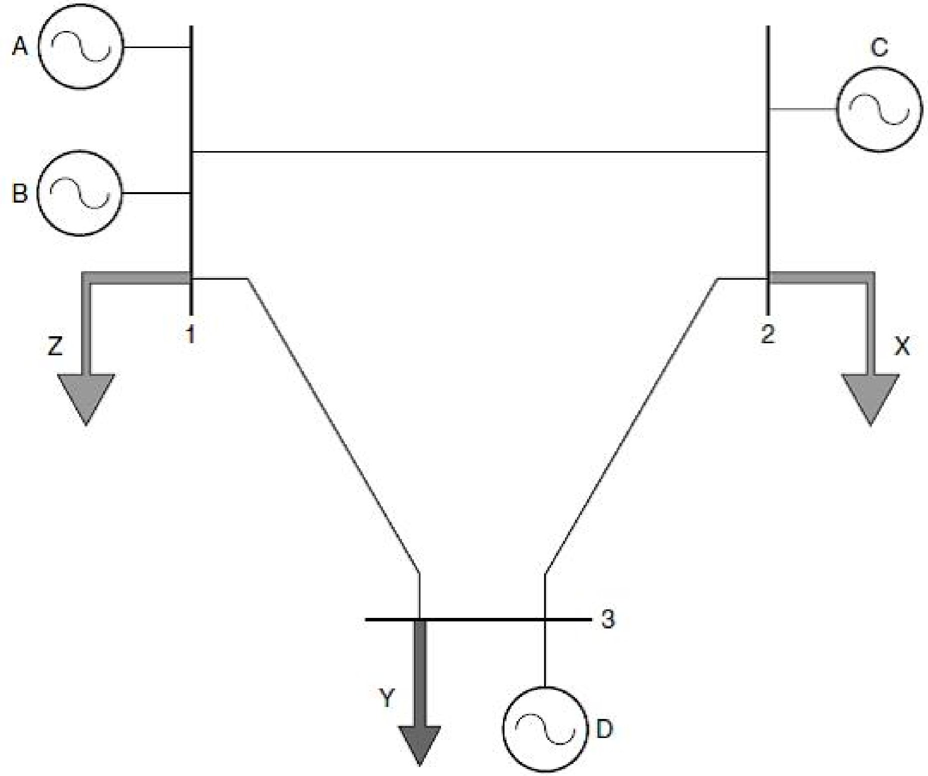
Let us suppose that generator B and load Y want to sign a contract for the delivery of 400 MW and that no other transactions occur. Is it possible? If not, what is the maximum amount of power they could trade?

Let path I be the made of the (oriented) links 1-2 and 2-3 and path II of the link 1-3.

$$F^{I} = \frac{0.2}{0.2 + 0.3} \times 400 = 160 \text{ MW}$$
$$F^{II} = \frac{0.3}{0.2 + 0.3} \times 400 = 240 \text{ MW}$$

Transaction not possible because path I has a maximum transfer capacity of 126 MW.

Maximum amount of power that can be traded: $\frac{0.5}{0.2} \times 126 = 315$ MW.



Branch	React. (p.u.)	Cap. (MW)
1-2	0.2	126
1-3	0.2	250
2-3	0.1	130

Let us suppose now that generator B and load Y still want to sign a contract for the delivery of 400 MW and that load Z also wants to purchase 200 MW from generator D. Is it possible?

Let (i) path III be the made of the (oriented) edges 3-2 and 2-1, (ii) path IV be made of the edge 3-1.

$$F^{III} = \frac{0.2}{0.2+0.3} \times 200 = 80\text{MW}$$
$$F^{IV} = \frac{0.3}{0.2+0.3} \times 200 = 120\text{MW}$$

Due to the superposition theorem, we can write:
$F_{12} = F_{23} = F^{I} - F^{III} = 160 - 80 = 80$ MW
$F_{13} = F^{II} - F^{IV} = 240 - 120 = 120$ MW

Transaction between generator D and load Z creates a counterflow that increases the power that Generator D and load Y can trade.

A few observations that can be drawn from this example:

1. The System Operator (SO) may reach false conclusions about the security of the system if not all proposed transactions are implemented.

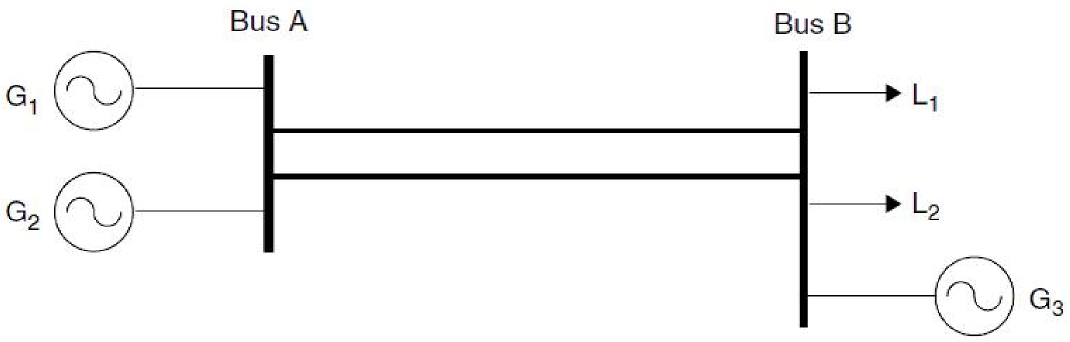
2. Difficult to organize a market for physical transmission rights because the transmission rights that can be sold depend on the use that the actors make of those rights.

3. Designing an efficient interaction process between the SO, the bilateral market and the market for physical transmission rights can be challenging.

Physical transmission rights and market power

In a perfectly competitive market, buying but not using them would be an irrational decision. However in a less perfectly competitive market, physical transmission rights can enhance the ability of some participants to exert market power.

Example: The most expensive generator (G_3) may want to secure the transmission rights between A and B to fend off competition from generators connected to A.



To avoid this problem, it has been suggested that a "use them or lose them" provision be attached to physical transmission rights.

Centralized trading over a transmission network

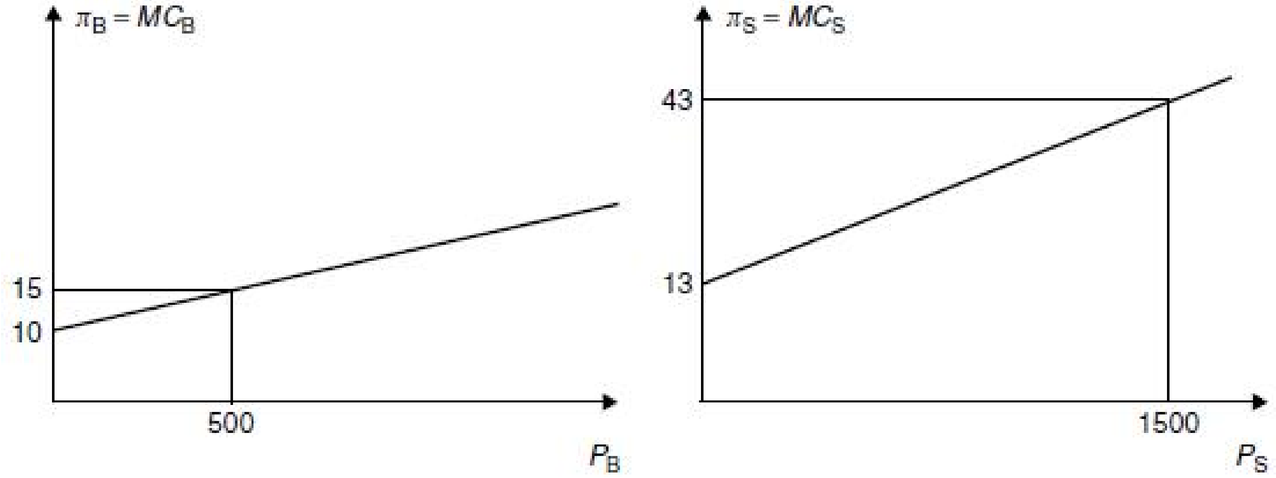
In centralized trading over a network, the system operator (or another that acts as market operator) selects the bid and the offers that optimally clear the market while respecting the security constraints imposed by the transmission network.

When losses or congestion taken into account, the price of electricity depends on the bus in which the power is injected or extracted.

Price that consumers and producers pay are the same for all participants connected to the same bus.

Centralized trading in a two-bus system

Supply function in two perfectly competititve markets, namely Borduria and Sylvadia):

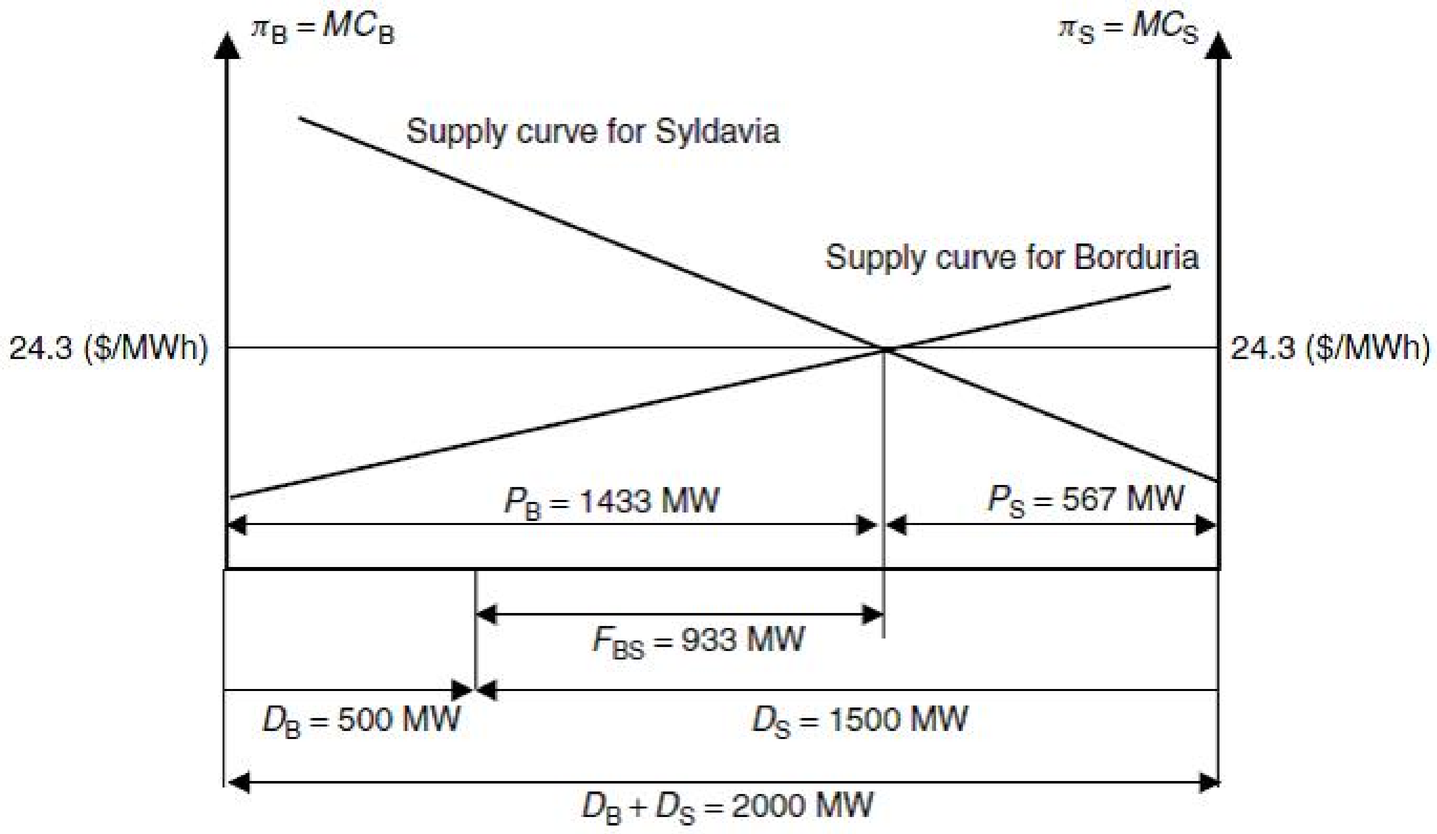


Equations of the supply curves: $\pi_B = MC_B = 10 + 0.01 P_B$ [\$/MWh]; $\pi_S = MC_S = 13 + 0.02 P_S$ [\$/MWh].

Demand in Borduria (D_B) and demand in Syldavia (D_S) constant and equal to 500 MW and 1500 MW, respectively.

Question: What will be the effects of connecting physically these two countries and coupling their electricity markets into a single one?

The unconstrained transmission case



Observations: (i) $\pi = \pi_B = \pi_S = 24.3$ [$/MWh], (ii) $F_{BS} = P_B - D_B = D_S - P_S = 933$ MW.

An optimisation formulation for finding the market dispatch:

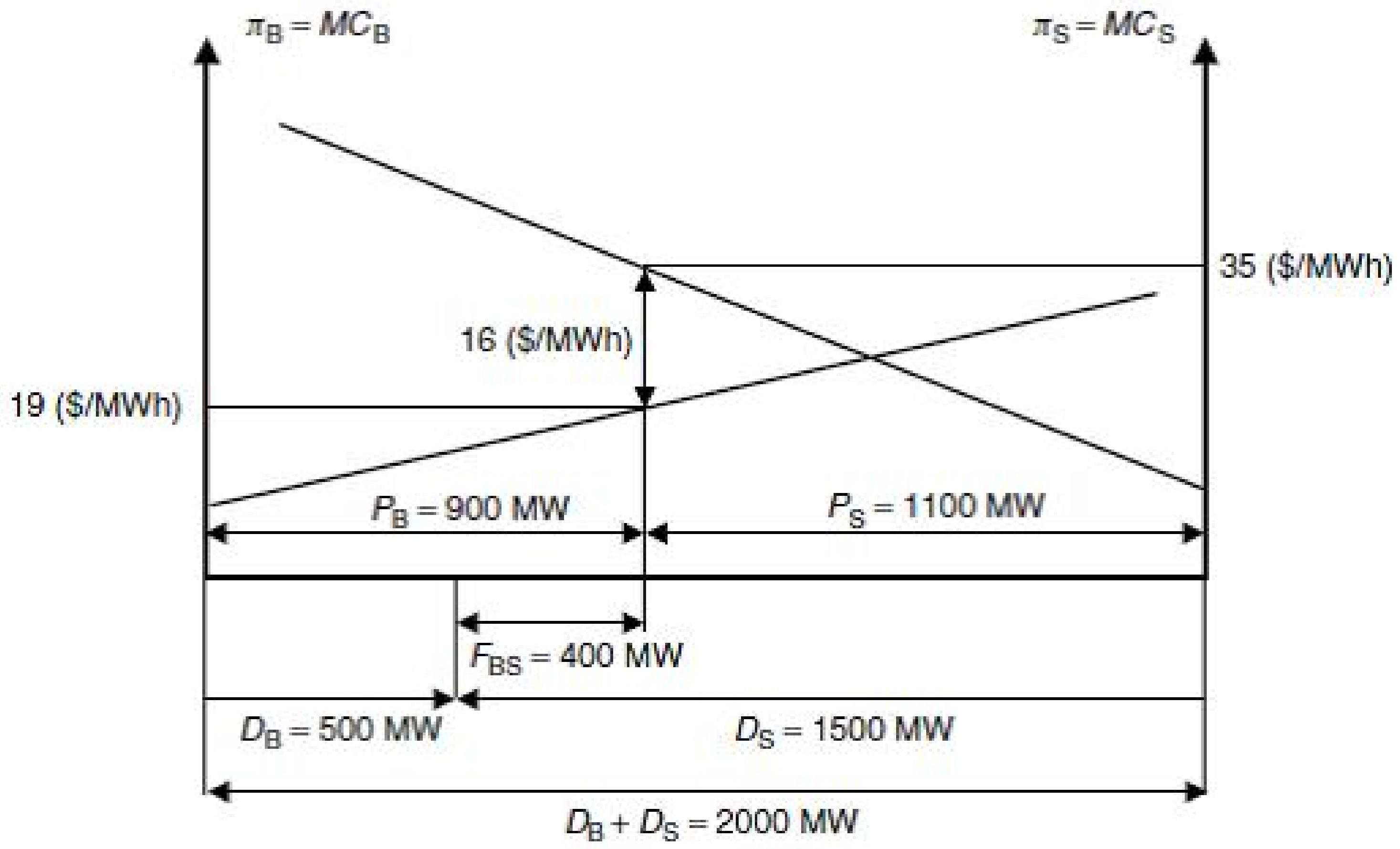
$$\min_{P_B,P_S} \int_{P_B} \pi_B(P_B)dP_B + \int_{P_S} \pi_S(P_S)dP_S$$

such that:

$$P_B + P_S = 1500$$

$$P_B, P_S \geq 0$$

The constrained transmission case - connection limited to 400 MW



Main observation: Marginal cost of production different in each country.

When the marginal cost depends on the location where the energy is produced or consumed, we talk about locational marginal prices.

If a different price is defined at each bus or node in the system, locational marginal prices are called nodal prices.

Locational marginal prices are usually higher in areas that normally import power and lower in areas that export power.

Loads pay the locational marginal price for every unit of energy consumed. Generators are paid the locational marginal price of every unit of energy produced.

An optimisation formulation for finding the market dispatch:

$$\min_{P_B, P_S} \int_{P_B} \pi_B(P_B) dP_B + \int_{P_s} \pi_S(P_S) dP_S$$

such that:

$$P_B + P_S = 1500$$

$$P_B, P_S \geq 0$$

$$P_B - D_B \leq 400$$

$$P_S - D_S \leq 400$$

Operation of the Borduria/Syldavia interconnection as separate markets, as a single market and as a single market with congestion (R = revenue accruing to a group of generators; E = payment made by a group of consumers):

	Separate markets	**Single market**	**Single market with congestion**
P_B (MW)	500	1433	900
π_B ($/MWh)	15	24.33	19
R_B ($/h)	7500	34 865	17 100
E_B ($/h)	7500	12 165	9500
P_S (MW)	1500	567	1100
π_S ($/MWh)	43	24.33	35
R_S ($/h)	64 500	13 795	38 500
E_S ($/h)	64 500	36 495	52 500
F_{BS} (MW)	0	933	400
$R_{TOTAL} = R_B + R_S$	72 000	48 660	55 600
$E_{TOTAL} = E_B + E_S$	72 000	48 660	62 000

Congestion surplus

Let us assume the following (i) Consumers pay their energy the going price in their local market, independently from where the energy they consume is produced (ii) The generators are paid the local price in their local market, independently from where the energy is consumed.

The merchandizing surplus is defined between the payments made by the loads and the revenues of the generators.

Question: What is the merchandizing surplus for the Borduria and Sylvadia example?

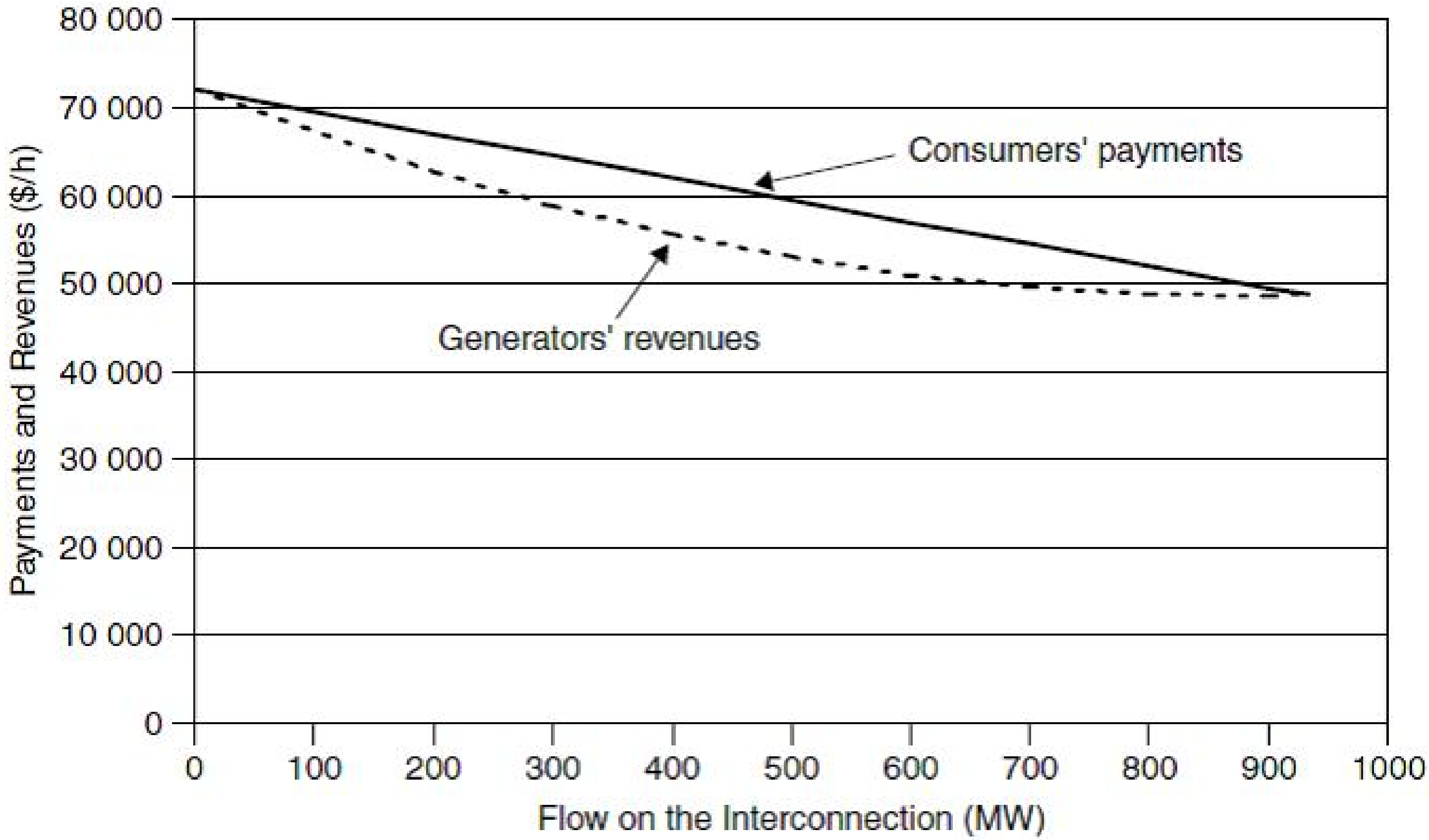
Electricity prices in Borduria and Syldavia:

$$\pi_B = MC_B = 10 + 0.01(D_B + F_{BS})$$
$$\pi_S = MC_S = 13 + 0.02(D_S - F_{BS})$$

Total payment: $E_{TOTAL} = \pi_B D_B + \pi_S D_S$.

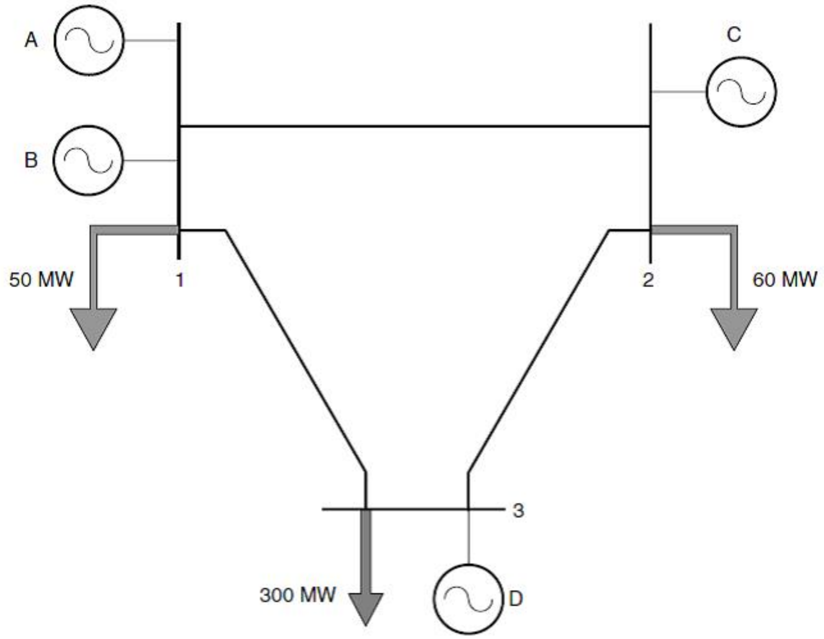
Total revenue: $R_{TOTAL} = \pi_B P_B + \pi_s P_S$.

Merchandizing surplus: $E_{TOTAL} - R_{TOTAL} = (\pi_S - \pi_B) F_{BS}$



The merchandizing surplus is also called the congestion surplus since it is due to congestion in the network.

Centralized trading in a three-bus system



Branch	React. (p.u.)	Cap. (MW)
1-2	0.2	126
1-3	0.2	250
2-3	0.1	130

Generator	Capacity (MW)	Marg. Cost ($/MWh)
A	140	7.5
B	285	6
C	90	14
D	85	10

Question: What is the optimal economic dispatch given the limits on the lines?

Step 1: Build the relation that relates the net power injected in every node of the network to the branch power flows, namely compute the elements F_{ij} defined as follows:

$$\begin{bmatrix} PB_{12} \\ PB_{23} \\ PB_{13} \end{bmatrix} = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

or $PB = F \times P$ in short, where P_i is the net power injected at bus i and PB_{ij} is the power flowing from i to j.

For computing these elements, you can assume values for the P_i (such as $\sum_i PB_i = 0$), compute the values of PB_{ij} (using the methodology seen previously) and write down the equations that the F_{ij}s must satisfy. Process needs to be repeated several times to get enough equations.

Example: Let $P_1 = 1$, $P_3 = -1$ and $P_2 = 0$. We have:
$PB_{12} = \frac{0.2}{0.2+0.3} = F_{11} - F_{13}$
$PB_{23} = \frac{0.2}{0.2+0.3} = F_{23} - F_{23}$
$PB_{13} = \frac{0.3}{0.2+0.3} = F_{31} - F_{33}$

Step 2: Solve the following optimization problem:

$$\min_{P_A,P_B,P_C,P_D,P_i,PB_i} 7.5P_A + 6P_B + 14P_c + 10P_D$$

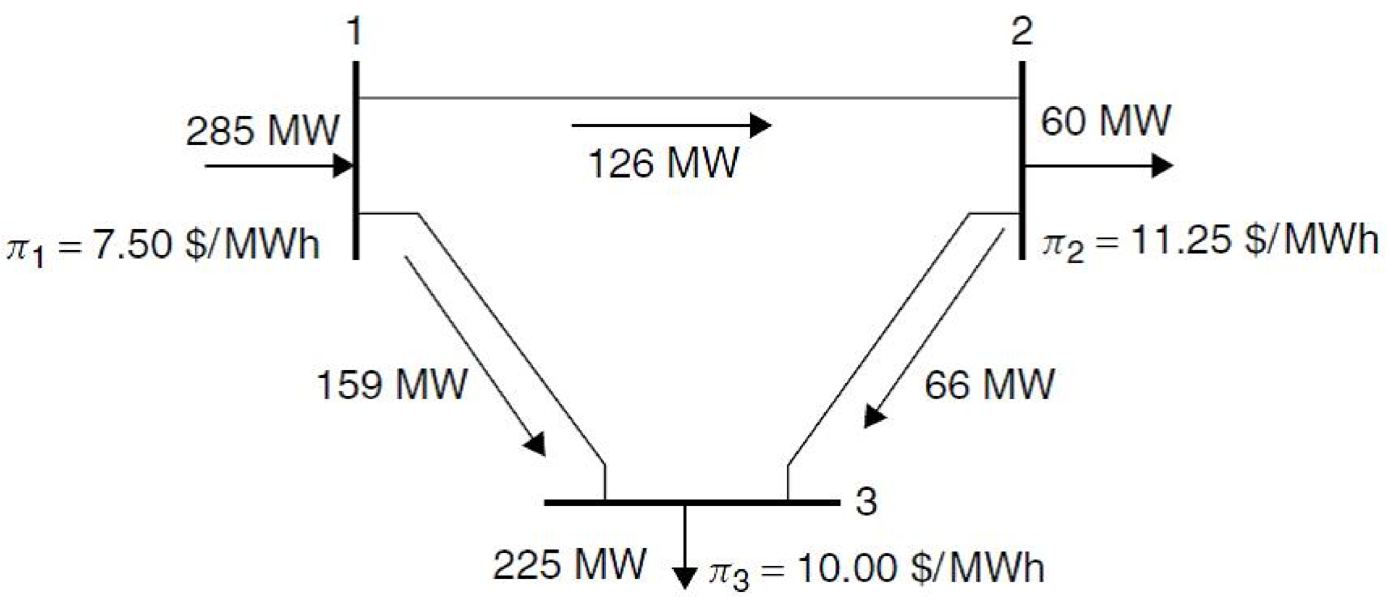
such that:

$$P_1 = P_A + P_B - 50$$
$$P_2 = P_C - 60$$
$$P_3 = P_D - 300$$
$$\sum_i P_i = 0$$
$$0 \leq P_A \leq 140$$
$$0 \leq P_B \leq 285$$
$$0 \leq P_C \leq 90$$
$$0 \leq P_D \leq 85$$
$$PB = F \times P$$
$$|PB_{12}| \leq 126$$
$$|PB_{13} \leq 250$$
$$|PB_{23}| \leq 230$$

Results

	Bus 1	Bus 2	Bus 3	System
Consumption (MW)	50	60	300	410
Production (MW)	335	0	75	410
Nodal marginal price ($/MWh)	7.50	11.25	10.00	–
Consumer payments ($/h)	375.00	675.00	3000.00	4050.00
Producer revenues ($/h)	2512.50	0.00	750.00	3262.50
Merchandising surplus ($/h)				787.50

Nodal marginal prices and power flows in the three bus system:



Contribution of each branch to the merchandising surplus of the three-bus system:

Branch	Flow (MW)	"From" price ($/MWh)	"To" price ($/MWh)	Surplus ($/h)
1-2	126	7.50	11.25	472.50
1-3	159	7.50	10.00	397.50
2-3	66	11.25	10.00	−82.50
Total				787.50

Two remarks:
(i) Economically couter-intuitive flows can happen, namely power can flow to nodes with lower marginal prices.
(ii) In our discussion we have assumed that the nodal markets are perfectly competitive. The nodal price is thus equal to the marginal cost when the energy is produced using local generators. However, a locational marginal price structure can make strategic bidding easy and profitable.

(iii) Nodal prices can run against good economic sense but they are mathematically correct in the sense that they maximize the global welfare.

Losses in transmission networks

Losses occur in electricity networks. Since one or more generators must produce this lost energy and since these generators expect to be paid for all the energy they produce, a mechanism must be devised to take losses and their cost into account in electricity networks.

Three types of losses: fixed losses, non technical losses and variable losses.

Fixed losses: Caused by (i) hysteresis and eddy current losses in the iron core of transformers (ii) corona effect in transmission lines. These losses are proportional to the square of the voltage and independent of the power flows and, as first approximation, can be considered as being constant.

Non technical losses: Energy which is stolen from the network.

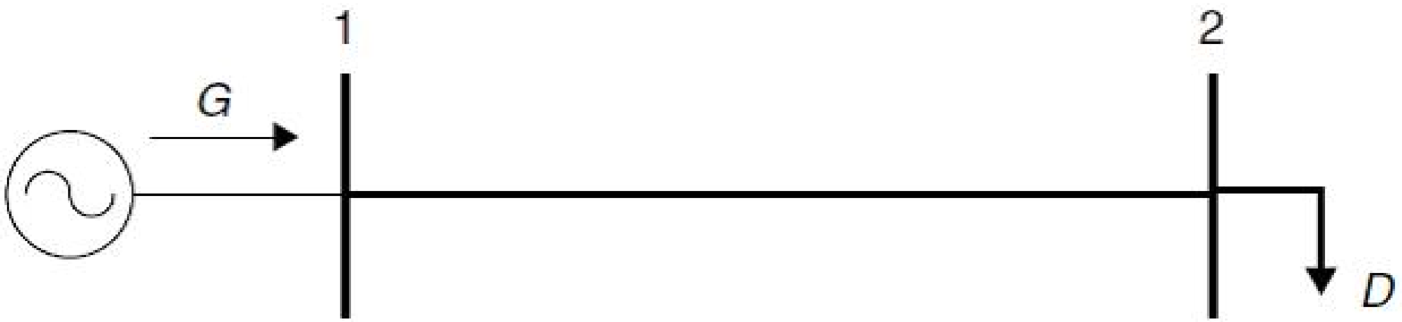
Variable losses: Also called tranported-related losses or copper losses. Proportional to te resistance R of the branch and to the square of the current in the branch. Since V does not deviate much from its nominal value and P much greater than Q, we have:

$$L^{variable} = I^2R \simeq (\frac{S}{V})^2R = \frac{P^2+Q^2}{V^2}R \simeq KP^2$$

Variable losses usually much higher than other losses. In western European countries, 1 to 3 % of the energy produced is lost in the transmission network and 4 to 9% in the distribution system.

Marginal cost of losses

Let us consider the following system:



Let $G(D) = D + L = D + KD^2$ be the power that has to be generated for covering load D and let us assume that the marginal cost of production is constant and equal to c.

Question: **(i)** Compute the locational marginal prices when taking into account losses **(ii)** Let the surplus be equal to the value of the energy sold at bus 2 minus the cost of purchasing the energy produced at bus 1. Show that the surplus is for this simple system equal to the cost of supplying the losses.

(i) If the load increases from D to $D + \Delta D$, the generation must increase by:
$\Delta G = G(D + \Delta D) - G(D) = \Delta D + 2\Delta DDK = (1 + 2DK)\Delta D$.
The corresponding increase in the cost of generation is:
$\Delta C = c(1 + 2DK)\Delta D$. We therefore have as locational marginal price at bus 2: $\pi_2 = \frac{\Delta C}{\Delta D} = c(1 + 2DK)$. The locational marginal price at bus 1 is $\pi_1 = c$.

(ii) Money paid by the load: $\pi_2 D = c(1 + 2DK)D$; Money received by the generator: $\pi_1(D + KD^2) = c(D + KD^2)$.
Surplus: $c(1 + 2DK)D - c(D + KD^2) = cKD^2$
Cost of losses: $c \times losses = cKD^2$

Handling losses under bilateral trading

Because losses are not a linear function of the flows in the transmission system, the losses caused by a transaction do not simply depend on the amount of power traded and the location of the two parties involved in the transaction. These losses also depend on all the other transactions taking place in the network.

Allocating the losses or their costs between all the market participants is thus a problem that does not have a rigorous solution.

A fair mechanism is one in which the participants that contribute more to losses pay a larger share than the others.

Homework

For a group of students, present a research paper which is related to the gaming of congestions in a power system.

For a group of students, present the part of the following paper: "Contract networks for electric power transmission" from W W Hogan that shows that minimizing the production costs when dealing with an inelastic demand is equivalent to maximizing the social welfare.

For a group of students, present the following paper: "Transmission loss allocation: a comparison of different practical algorithms" from AJ Conejo.