ELEC0018-1 – Energy Markets

Workshop 2 : day-ahead market (1)

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Context

- One day before delivery time.
- Adapt your position for the 24 hours of the next day.
- Producers want to sell energy if they expect to make a profit (they are said to be **in the market**) or want to buy the energy they have to deliver if it is less expensive than producing it by themselves (**out of the market**).
- Retailers want to buy energy for their consumers. They buy or sell depending on if they expect a surplus or a shortage depending on their forward contracts.

Day-ahead market (EPEX SPOT)

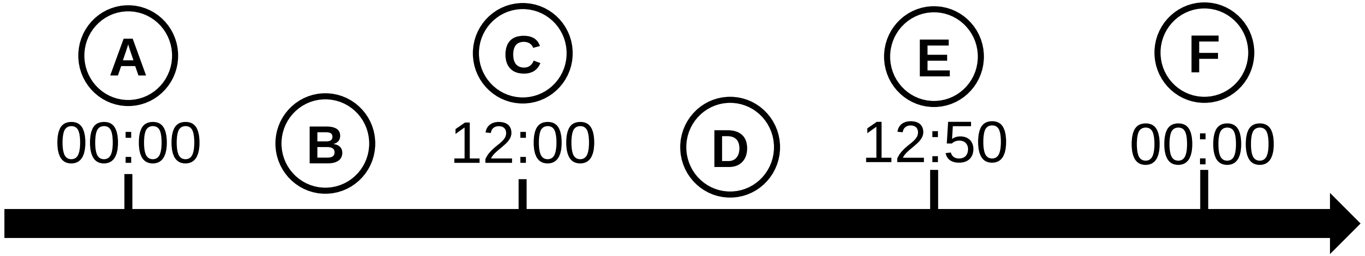
- Also called electricity **spot** market.

- Pool market for buying energy : the agents can **submit offers** during the opening hours and the market is cleared after closure.

- **Hourly products** : buy or sell electricity for each hours of the next day.

- Market operator: EPEX SPOT (originally Belpex).

Day-ahead market – Timeline



A. Opening of the day-ahead market for all hours of the following day.
B. Market participants submit their bids and asks to the order book (simple orders, block orders, exclusive orders, curtailable orders, ...).
C. Closing of the day-ahead market for all hours of the following day.
D. Execution of the market clearing algorithm.
E. Notification of the market participants and system operators about the market clearing outcomes.
F. Beginning of the delivery of electricity for the entire day.

Terminology

- On the day-ahead market, we trade **energy**. Nevertheless, we often speak about capacity, we mean this capacity for one hour.

- An offer for buying or selling energy is called an **order or a bid**.

- An order for selling energy is called a **sell** order or a **bid** order.

- An order for buying energy is called a **buy** order or an **ask** order.

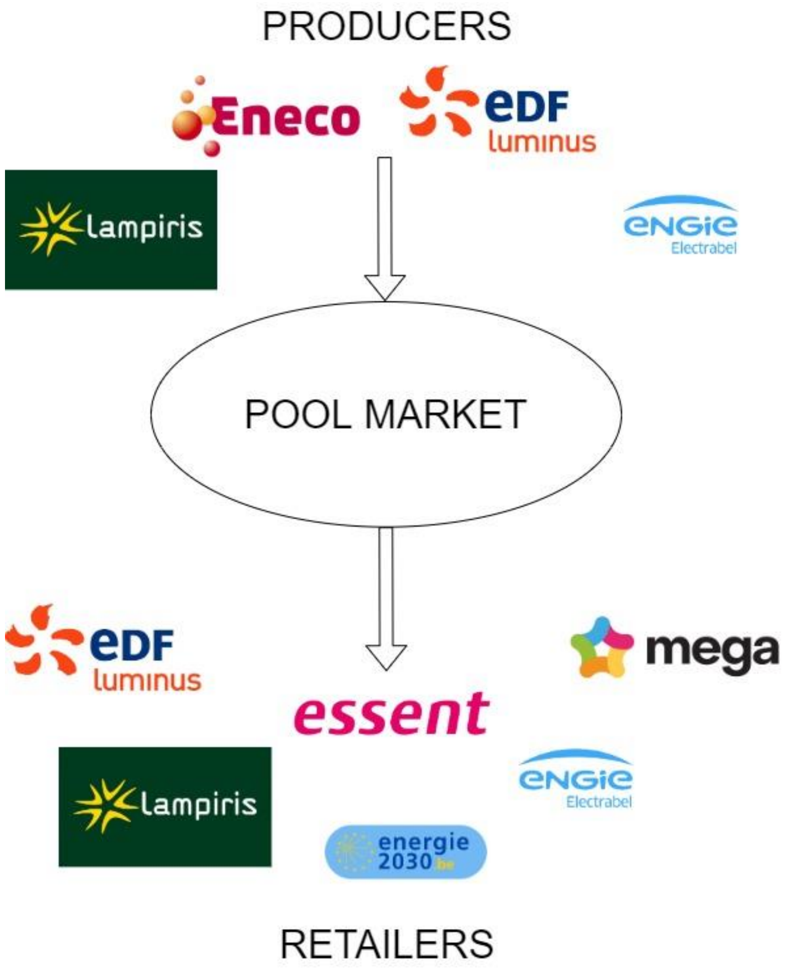
Participating in the market

- Agents can submit bids/offers in the pool
  Bid = sell/buy + price + quantity + hour

- The bids represent the **willingness to pay** or to be payed. How much does an agent valorizes the energy.

- Bids are **anonymous**.

- Producers and retailers do not interact aside of the market even if they are from the same firm. All bidding decisions must be based on public information !



Market clearing - Exercice

- Assuming you are the market operator and you see the bids in the pool :
  - How much shall a consumer pay at least for buying 20 MWh? What is the cost of the last unit of energy?
  - How much will a producer get paid at most for selling 20 MWh? What is the cost of the last unit of energy?
  - More generally for x MWh ?

Sell	50 MWh	20 €/MWh
	100 MWh	10 €/MWh
	20 MWh	30 €/MWh
	200 MWh	5 €/MWh
	10 MWh	0 €/MWh
Buy	50 MWh	1 €/MWh
	100 MWh	15 €/MWh
	200 MWh	20 €/MWh
	50 MWh	30 €/MWh

Market clearing - Exercice

- Assuming you are the market operator and you see the bids in the pool :
  - How much shall a consumer pay at least for buying 20 MWh?

$$10\text{MWh} * 0€/\ \text{MWh} + 10\text{MWh} * 5€/\ \text{MWh} = 50€$$

  - What is the cost of the last unit of energy?

$$5€$$

Marginal cost of energy for a consumer

Sell	50 MWh	20 €/MWh
	100 MWh	10 €/MWh
	20 MWh	30 €/MWh
	200 MWh	5 €/MWh
	10 MWh	0 €/MWh
Buy	50 MWh	1 €/MWh
	100 MWh	15 €/MWh
	200 MWh	20 €/MWh
	50 MWh	30 €/MWh

Market clearing - Exercice

- Assuming you are the market operator and you see the bids in the pool :
  - How much will a producer get paid at most for selling 20 MWh?

$$20\text{MWh} * 30€/\text{ MWh} = 600€$$

  - What is the cost of the last unit of energy?

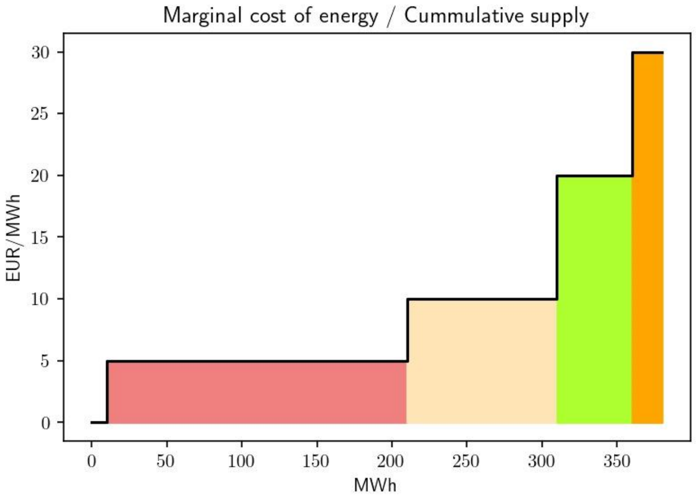
$$30€$$

Marginal revenue of energy for a producer

Sell	50 MWh	20 €/MWh
	100 MWh	10 €/MWh
	20 MWh	30 €/MWh
	200 MWh	5 €/MWh
	10 MWh	0 €/MWh
Buy	50 MWh	1 €/MWh
	100 MWh	15 €/MWh
	200 MWh	20 €/MWh
	50 MWh	30 €/MWh

Market clearing - Exercice

- Assuming you are the market operator and you see the bids in the pool :
  - Marginal cost for x MWh ?



Sell	50 MWh	20 €/MWh
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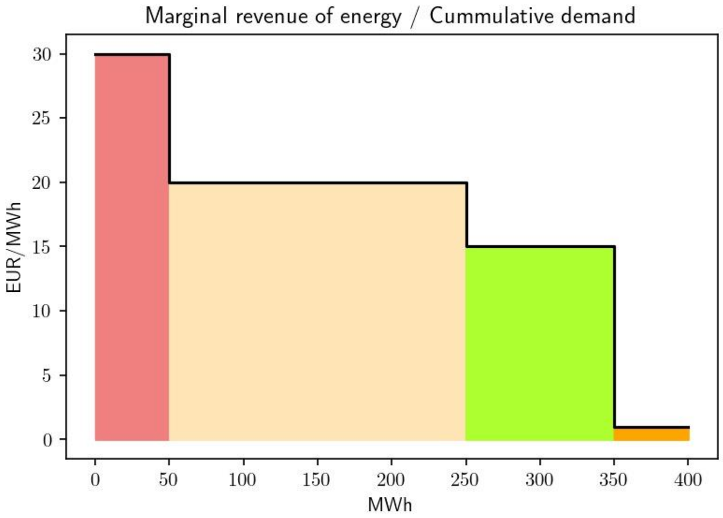
	10 MWh	0 €/MWh
Buy	50 MWh	1 €/MWh
	100 MWh	15 €/MWh
	200 MWh	20 €/MWh
	50 MWh	30 €/MWh

Market clearing - Exercice

- Assuming you are the market operator and you see the bids in the pool :
  - Marginal revenue for x MWh ?



Sell	50 MWh	20 €/MWh
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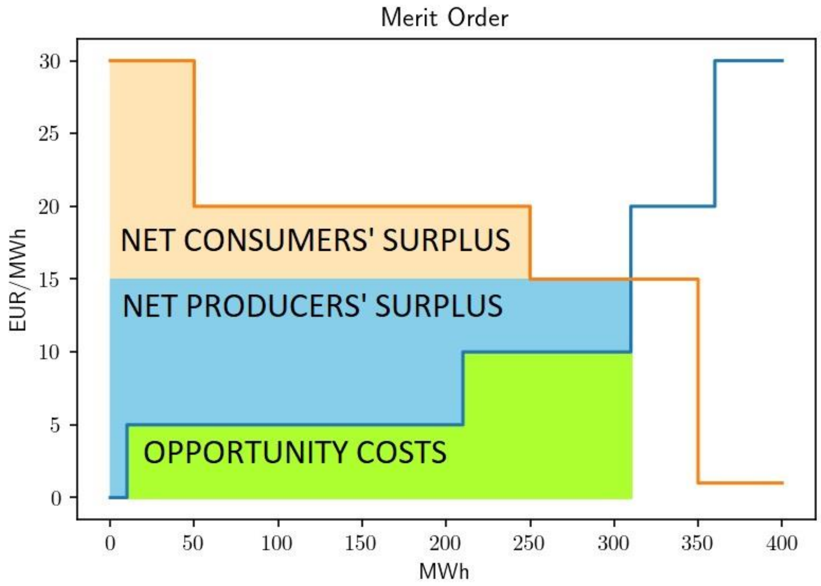
	200 MWh	5 €/MWh
	10 MWh	0 €/MWh
Buy	50 MWh	1 €/MWh
	100 MWh	15 €/MWh
	200 MWh	20 €/MWh
	50 MWh	30 €/MWh

Merit Order

- **Merit Order** : ordering of the bids.
  - The marginal **cost** represents the cumulative **supply** curve.
  - The marginal **revenues** represents the cumulative **demand**.
- **Social welfare** : area between supply and demand curves. It equals to the sum of the net consumers’ suplus and the net producers’ suplus.

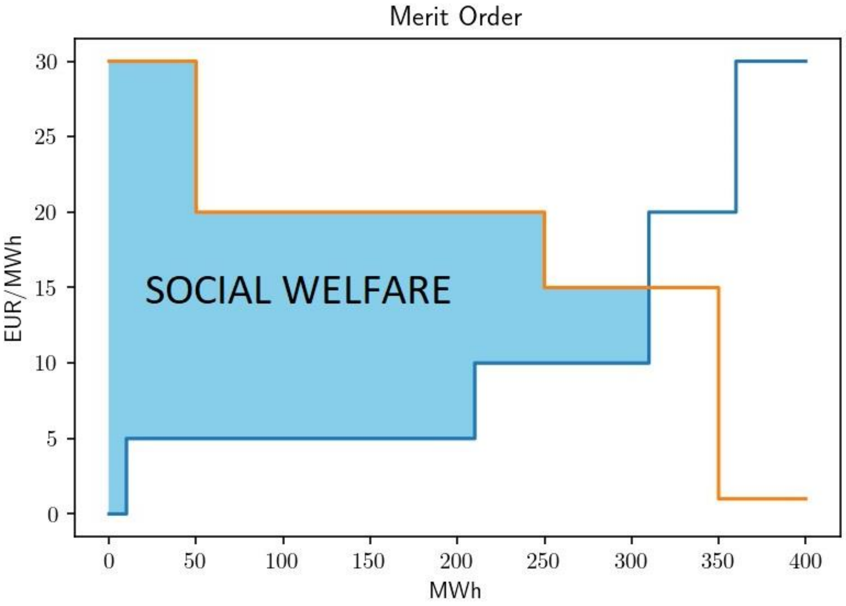


Market clearing - Merit Order

- **Equilibrium price** p_{eq} : intersection of supply and demand.

- The social welfare represents the ‘benefit of the clearing if paid at the equilibrium price’.

- The objective of the market operator is to clear (accept) the bids so as to **maximize the social welfare**.

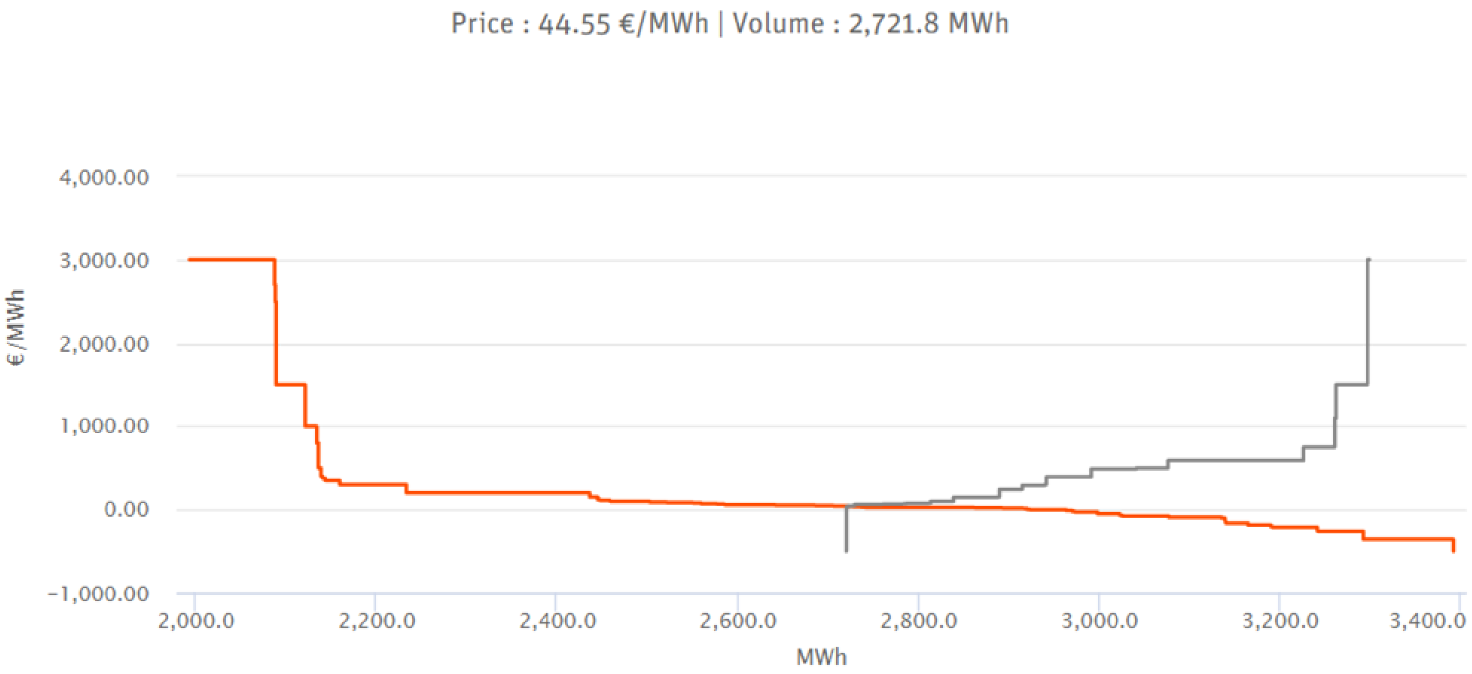


Market clearing - Settlement

What is the final cost of electricity?

- Two paying mechanisms :
  - **Paid-as-bid** : each agent receives the amount of money they bid.
  - **Uniform pricing** : a single price is fixed as the market price.
- EPEX : uniform pricing at the equilibrium price.
- With uniform pricing, the agents have the incentive to bid at their marginal cost.

EPEX auction - 03/09/2020 product of 00h00-01h00



https://www.epexspot.com/en/market-data?market_area=BE&trading_date=2020-09-02&delivery_date=2020-09-03&underlying_year=&modality=Auction&sub_modality=DayAhead&product=60&data_mode=aggregated&period=


Price
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Hours

Optimization problem - Overview

Assumptions:

- Given a set of bids for a given hour of the day.
- A bid can be partially cleared. We can thus buy a portion of the quantity bid at the given price.
- Uniform pricing at the equilibrium price p_{eq}.

Objectives:

- Maximizing the social welfare.
- Demand shall equals supply.

Optimization problem - Formulation

- Generation bids:
  - N_G generation bids : set of offers $L_G = \{ G_i | i = 0, \ldots, N_G - 1\}$
  - Quantity for offer G_i : E_i^G
  - Price for offer G_i : p_i^G

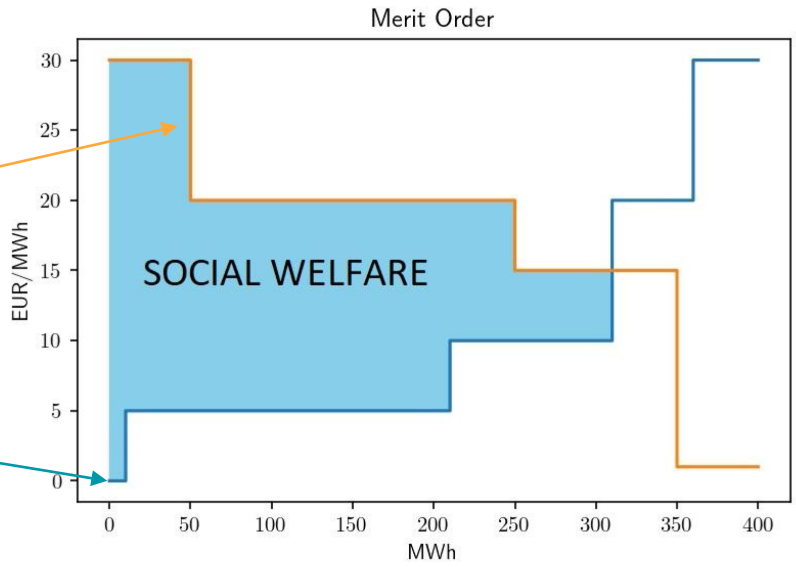
- Demand bids:
  - N_D demand bids : set of offers $L_D = \left\{ D_j \middle| j = 0, \ldots, N_D - 1\right\}$
  - Quantity for offer D_j : E_j^D
  - Price for offer D_j : p_j^D

- Variables of the problem :
  - Generation level y_i^G of offer G_i : how much energy is cleared from the bid G_i at price p_i^G
  - Consumption level y_j^D of offer D_j : how much energy is cleared from the bid D_j at price p_j^D

Optimization problem – Objective

- Objective : finding the **dispatch** $\{y_i^{G*}\},\{y_j^{D*}\}$ maximizing the social welfare, i.e. area between the supply and demand bids cleared. This area is oriented positive if the demand price is greater than the supply price.

$$\max_{\{y_i^G\},\{y_j^D\}} \sum_{j=0}^{N_D} p_j^D y_j^D - \sum_{i=0}^{N_G} p_i^G y_i^G$$



- This equation is only valid if the cumulative demand equals the cumulative supply !

Optimization problem – Constraints

- Supply (generation) equals demand (consumption) :

$$\sum_{i=0}^{N_G} y_i^G - \sum_{j=0}^{N_D} y_j^D = 0$$

- A bid can be **partially cleared** but we cannot accept less energy than zero from a bid or more than the quantity proposed at the given price :

$$0 \leq y_i^G \leq E_i^G \quad , \forall \, i = 0, \ldots, N_G - 1$$
$$0 \leq y_j^D \leq E_j^D \quad , \forall \, j = 0, \ldots, N_D - 1$$

Optimization problem

$$\max_{\{y_i^G\},\{y_j^D\}} \sum_{j=0}^{N_D} p_j^D y_j^D - \sum_{i=0}^{N_G} p_i^G y_i^G$$

$$subject\ to \qquad \sum_{i=0}^{N_G} y_i^G - \sum_{j=0}^{N_D} y_j^D = 0$$

$$0 \le y_i^G \le E_i^G \quad ,\forall\, i = 0, \dots, N_G - 1$$

$$0 \le y_j^D \le E_j^D \quad ,\forall\, j = 0, \dots, N_D - 1$$

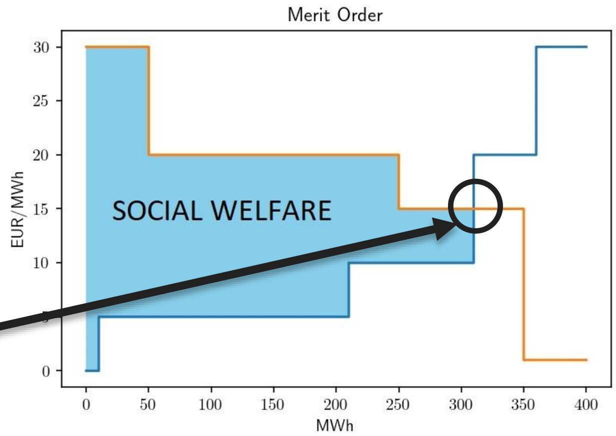
- The solution to the problem is the **optimal dispatch** for the market participant : the sequence of generation and consumption levels $\{y_i^{G*}\},\{y_j^{D*}\}$ of each offer maximizing the social welfare.

Optimization problem - Settlement

- In order to have the complete market-clearing, the **system price** shall be known in addition to the optimal dispatch.

- The equilibrium price p_{eq} is the marginal price of electricity at the point where the supply equals the demand. It corresponds to the influence of a small variation ε of the equality between supply and demand on the objective function.

$$\Delta\left(\sum_{i=0}^{N_G} y_i^G - \sum_{j=0}^{N_D} y_j^D\right) = \varepsilon$$



- Finding the marginal costs of variations of the constraints on the objective is achieved solving the so-called **dual optimization** problem.

Linear programming – Dual problem

- Solving the dual problem consists in finding the **smallest upper bound** on the objective function that respects the constraints.

- In linear programming it is achieved as follows:
    1. For each constraint, define a dual variable.
    2. Multiply the constraints by their respective dual variables and sum them up.
    3. Upper bound the coefficients of the objective function with the coefficients of the equation from point 2.
    4. The signs of the dual variables are such that the constant term (w.r.t. the primal variables) of the equation from point 2 upper bounds the objective function.
    5. The smallest upper bound in function of the dual variables equals the optimal primal objective (strong duality in linear programming).

Dual clearing problem - Formulation

1. Let v_{eq}, v_i^G and v_j^D be the dual variables corresponding to the equality constraint, to the inequalities $\mathrm{y}_i^G \leq E_i^G$ and to the inequalities $\mathrm{y}_j^D \leq E_j^D$, respectively.
2. Let's linearly combine the constraints with $v_i^G \geq 0$ and $v_j^D \geq 0$:

$$v_{eq}\left(\sum_{i=0}^{N_G} y_i^G - \sum_{j=0}^{N_D} y_j^D\right) + \sum_{i=0}^{N_G} v_i^G y_i^G + \sum_{j=0}^{N_D} v_j^D y_j^D \leq \sum_{i=0}^{N_G} v_i^G E_i^G + \sum_{j=0}^{N_D} v_j^D E_j^D$$

3. We want the lhs to be an upper bound on the objective:

$$\sum_{j=0}^{N_D} p_j^D y_j^D - \sum_{i=0}^{N_G} p_i^G y_i^G \leq v_{eq}\left(\sum_{i=0}^{N_G} y_i^G - \sum_{j=0}^{N_D} y_j^D\right) + \sum_{i=0}^{N_G} v_i^G y_i^G + \sum_{j=0}^{N_D} v_j^D y_j^D$$

Since the primal variables are positive, it implies that:

$$v_i^G + v_{eq} \geq -p_i^G$$
$$v_j^D - v_{eq} \geq p_j^D$$

Dual clearing problem - Formulation

4. The signs of the dual variables were chosen so as to have the lhs smaller than the rhs in the equation from point 2. Doing so, we have an upper bound on the objective function.
5. The dual objective is to minimize the upper bound :

$$\min_{v_{eq},\{\mathrm{v}_i^G\},\{\mathrm{v}_j^D\}} \sum_{i=0}^{N_G} v_i^G E_i^G + \sum_{j=0}^{N_D} v_j^D E_j^D$$

$$\begin{aligned} \textit{subject to} \quad & v_i^G + v_{eq} \geq -p_i^G \\ & v_j^D - v_{eq} \geq p_j^D \\ & v_i^G \geq 0 \\ & v_i^D \geq 0 \end{aligned}$$

Dual clearing problem – Dual variables

- For the optimal dispatch and the optimal dual variables, we have that the upper bound of the equation point 2 equals the maximal social welfare :

$$\sum_{j=0}^{N_D} p_j^D {y_j^D}^* - \sum_{i=0}^{N_G} p_i^G {y_i^G}^* = v_{eq}^* \left(\sum_{i=0}^{N_G} {y_i^G}^* - \sum_{j=0}^{N_D} {y_j^D}^* \right) + \sum_{i=0}^{N_G} {v_i^G}^* {y_i^G}^* + \sum_{j=0}^{N_D} {v_j^D}^* {y_j^D}^*$$

- By how much will the social welfare increase for a variation ε of the equality constraint ?

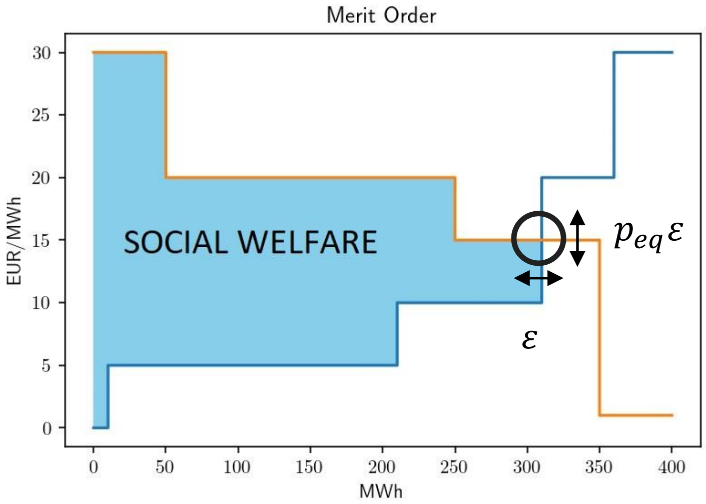
$$\Delta \left(\sum_{i=0}^{N_G} y_i^G - \sum_{j=0}^{N_D} y_j^D \right) = \varepsilon$$

Dual clearing problem – Dual variables

- If we change by ε the equality constraint, the equations tell us that the social welfare will change by $v_{eq}^{*}\varepsilon$.

- Graphically, a change in the social welfare at the optimum corresponds to a change of area with width ε and with height $p_{eq}\varepsilon$.

- It follows that $v_{eq}^{*} = p_{eq}$ is the cost for producing the most expensive unit of electricity in the merit order. It is the **clearing price** !



Linear programming

- The optimization problem for finding the optimal dispatch is linear. It can thus be written as follows (**primal** form):

$$
\begin{aligned}
\max_{y} \quad & c^T y & & \\
s.t. \quad & A_{eq}^T\, y & = \quad & b_{eq} \\
& A^T y & \leq \quad & b \\
& y & \geq \quad & 0
\end{aligned}
$$

- Its **dual** form writes as:

$$
\begin{aligned}
\min_{v} \quad & v^T b & & \\
s.t. \quad & A_{eq}\, v_{eq} + A\, v & \geq \quad & c \\
& v & \geq \quad & 0
\end{aligned}
$$

Inelastic demand - Overview

- What if there is a demand D that **must be delivered** whatever the price?

- The social welfare is infinite since the consumer is ready to pay an infinite price in the worst case. It is only infinite since the **net consumer' surplus is infinite**.

- In this context, maximizing the social welfare corresponds to maximizing the net producers' surplus. Equivalently, the optimal dispatch is the one that minimizes the area under the cleared generation bids, this area is called the **opportunity cost**.

$$\min_{\{y_i^G\}} \quad \sum_{i=0}^{N_G} p_i^G y_i^G$$
$$s.t. \quad \sum_{i=0}^{N_G} y_i^G = D$$
$$0 \leq y_i^G \leq E_i^G$$

Inelastic demand – In practice

- In practice, it might happen that all the supply does not cover the demand. The previous optimization problem has no solution.

- We can add to the objective the participation of the cost p_{shed} for the part of the demand ΔD that is not supplied. This price corresponds to the **Value Of Loss Load** (VOLL) and is usually set to $p_{shed} = 1000$€/MWh.

$$\min_{\{y_i^G\},\Delta D} \quad \sum_{i=0}^{N_G} p_i^G y_i^G + p_{shed}\,\Delta D$$

$$s.t. \quad \sum_{i=0}^{N_G} y_i^G + \Delta D = D$$

$$0 \le y_i^G \le E_i^G$$

$$0 \le \Delta D \le D$$

Negative market prices

- At how much do we bid the nuclear capacity and the RES?

- What is the marginal cost of stopping a nuclear plant?

- Do we stop a wind farm from producing? The energy is available whatever the demand? How does the government policies keeps it interesting for wind plant to produce?

    - **Premium support** : RES producers receive a fix support on top of the market price.

    - **Feed-in tariff support** : RES producers are paid a guaranteed price if the market price is below it, otherwise they receive the market price.

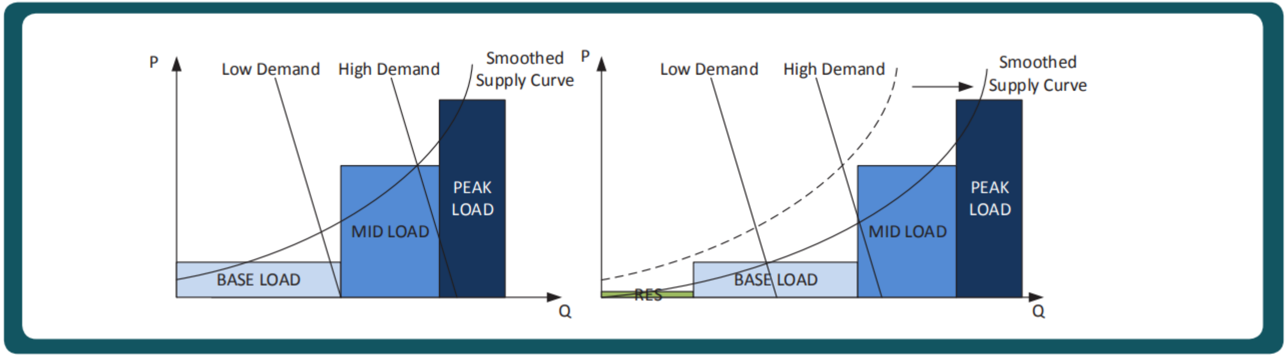


Figure 1: Theoretical merit order without (left) and with renewable energy sources (right)

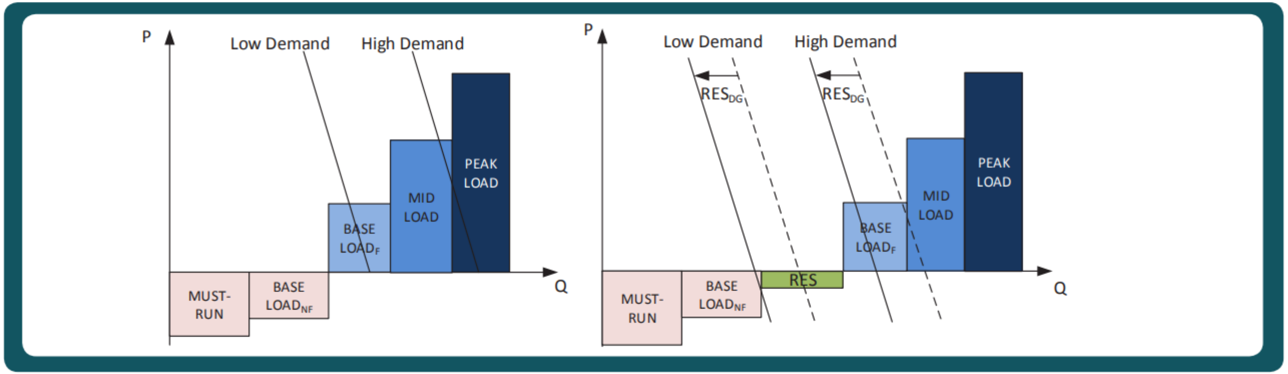


Figure 2: Practical merit order without (left) and with renewable energy sources (right); RES_{DG} expected renewable generation production of distributed nature; F flexible; NF non-flexible

https://set.kuleuven.be/ei/images/negative-electricity-market-prices

References

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