Power sharing in DC microgids

Optimization and optimal power flow

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Agenda

- A quick overview of mathematical programming
    - A focus on Second Order Cone Programming (SOCP)
- The formulation of optimal power flow OPF in a DC microgrid as a NLP
    - Relaxation as a SOCP
- Hands on session with Python and Pyomo to model the OPF formulations

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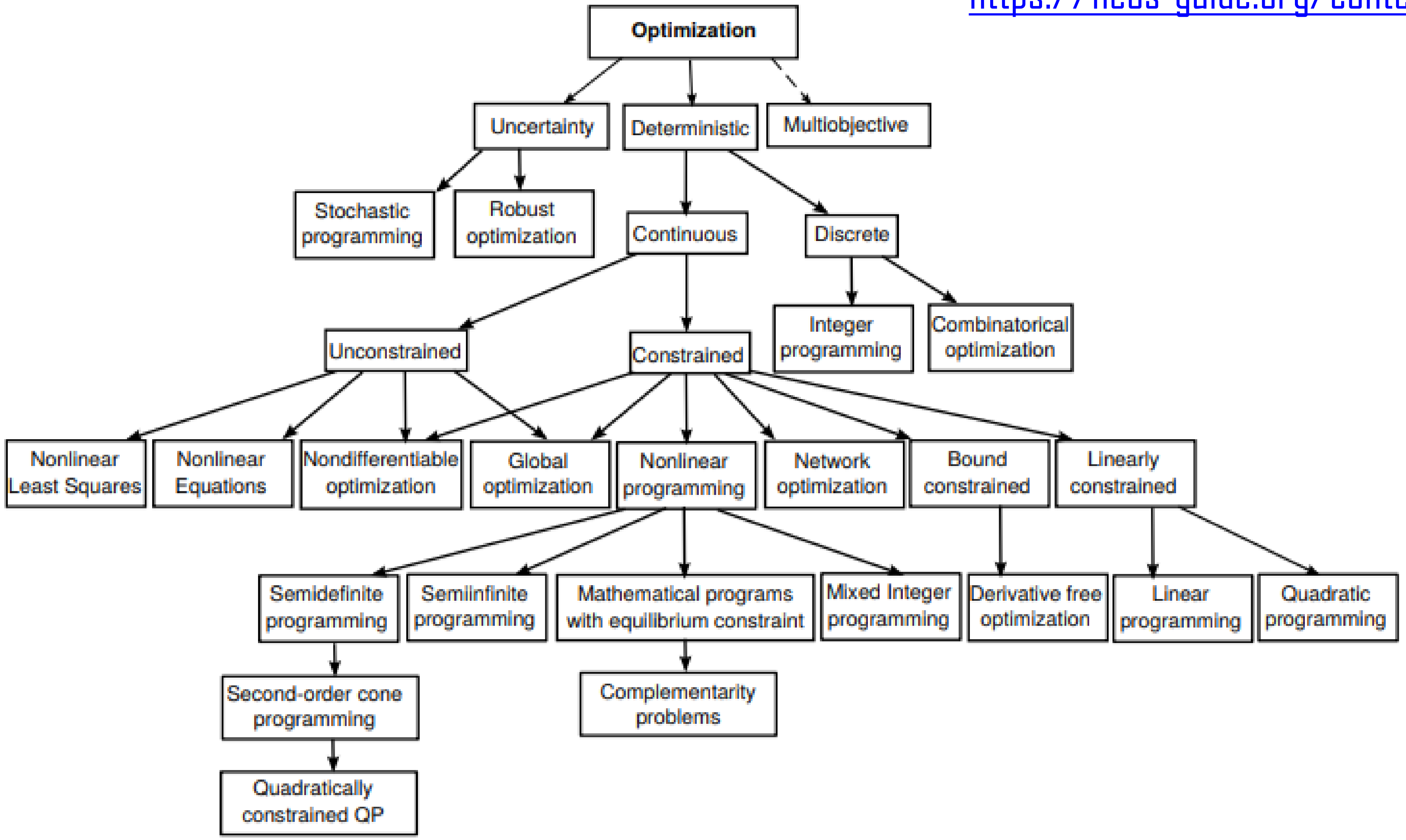
Mathematical Programming

- A field of applied mathematics that deals with the solution of **optimization problems**
- A framework and solution methods for computing the decisions of an optimization problem, given an **objective** function to minimize or maximize, and (optionally) **constraints** on the **decision variables**
- Mathematical programming relies on a **model** of the problem to solve
- A **great variety of mathematical programming problem types** depending on
  - the characteristics of the objective function
  - the characteristics of the constraints
  - the restrictions that apply to variables

Classification of Optimization problems

https://neos-guide.org/content/optimization-taxonomy



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The main categories we are interested in

General mathematical program

A general mathematical program can be stated as follows:

$$\begin{aligned} &\min\ f(x) \\ &s.t.\ g(x) \leq 0 \\ &\qquad Ax = 0 \\ &\qquad x \in X \end{aligned}$$

It is very hard to solve, especially when

- objective and constraints are non-linear or even worse non-convex
- variables are discrete

or continuous

Linear program

$$\begin{aligned} &\min\ c^T x \\ &s.t.\ Ax = b \\ &\qquad x \in \mathbb{R}^n_+ \end{aligned}$$

Easy to solve even for large problems.

Mixed-Integer Linear program

$$\begin{aligned} &\min\ c^T x \\ &s.t.\ Ax = b \\ &\qquad x \in \mathbb{R}^{n_1}_+ * \mathbb{Z}^{n_2}_+ \end{aligned}$$

Hard problem, but feasible for moderately sized instances.

Linear programming and MILP

- For an intro to linear programming LP and Mixed Integer Linear Programming (MILP), see https://bcornelusse.github.io/ELEN0445-microgrids/pdf/intro_math_programming-v1.pdf
  - video about LP
  - video about MILP

Nonlinear programming

- In a general non-linear (non-convex) problem, there is no guarantee that a locally optimal solution is also a global solution
- When there are no discrete variable, you can solve them (locally) with interior point methods
    - E.g. solver: ipopt

Convex programming

See the book "Stephen P. Boyd, and Lieven Vandenberghe. Convex optimization. Cambridge university press, 2004."
Freely available, you can read the introduction section

- We minimize a convex function over a convex set
- Convexity of a function f means
  - $f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x) + (1-\lambda) f(x_2) \ \ \forall \lambda \in [0,1]$
- Convexity of a set F means
  - If $x \in F$ and $y \in F$
  - then $\lambda x + (1-\lambda) y \in F, \ \forall \lambda \in [0,1]$

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Why convex optimization

- Convexity guarantees that a local optimal solution is also global
- For some special cases, very efficient algorithms exist
- Use it to solve nonconvex problems
  - For initialization
  - To get some bounds for global optimization
    - Convex relaxation

Second-Order Cone Programming

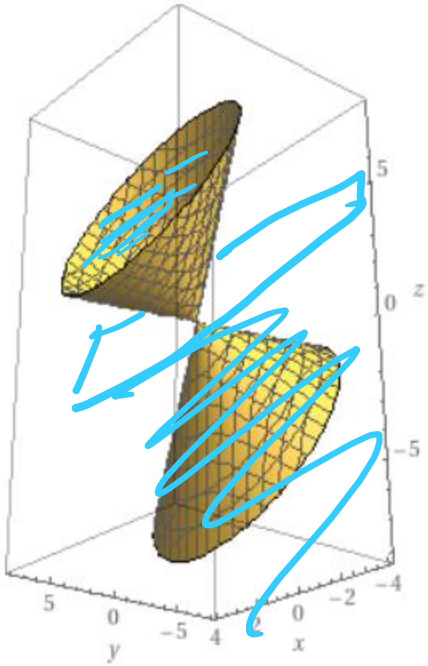
- A class of convex problems that is more general than LP but for which efficient algotihms exist

- In this problem, some constraints define cones:

  - A set C is called a cone if for every $x \in C$ and $\theta \geq 0$ we have $\theta x \in C$.

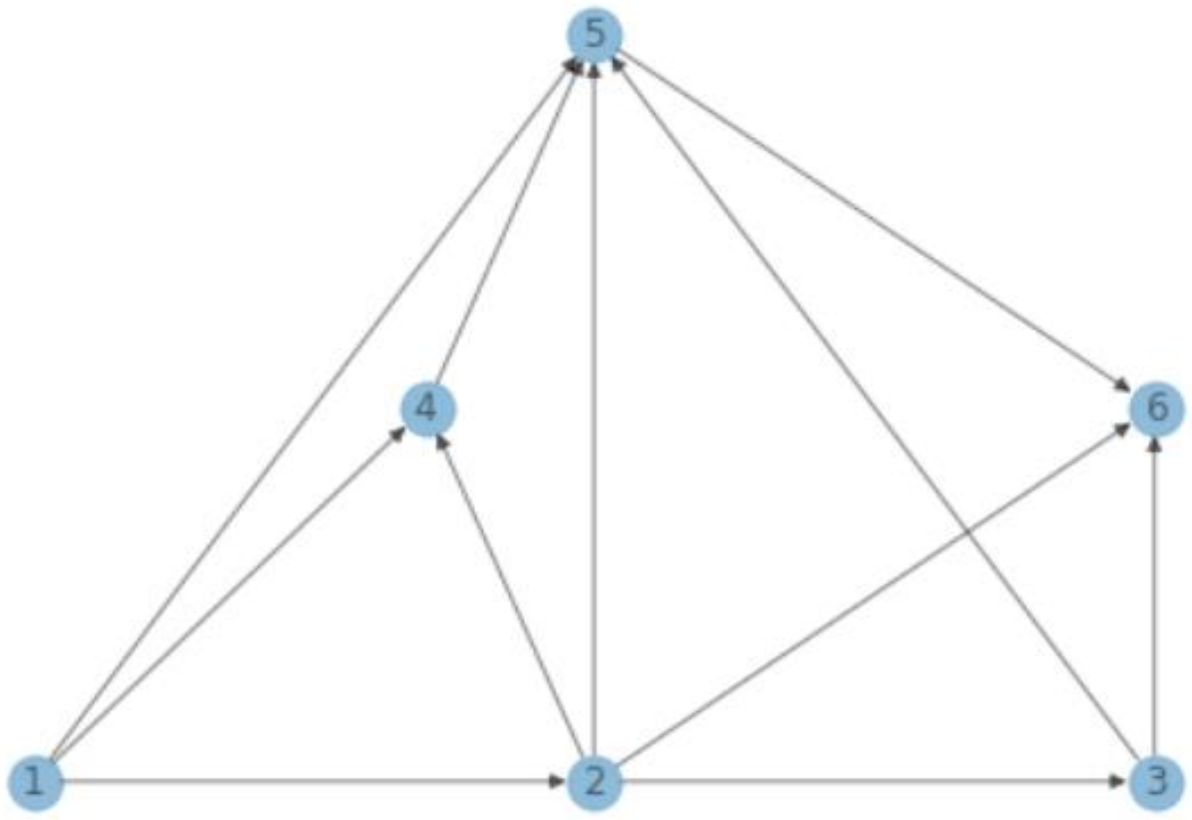
Examples

elliptic cone



- $\{(x, y, z) \mid x^2 \leq yz\}$
- $\{(x_1, x_2, t) \mid (x_1^2 + x_2^2)^{1/2} \leq t\}$

Optimal power flow



Gan, L., & Low, S. H. (2014). Optimal power flow in direct current networks. *IEEE Transactions on Power Systems*, *29*(6), 2892–2904.

Problem formulation

$$\begin{aligned}
\textbf{OPF}: \ \min \ & \sum_{i \in \mathcal{N}} f_i(p_i) \\
\text{over } & p, V \\
\text{s.t. } & p_i = V_i \sum_{j : j \sim i} (V_i - V_j) y_{ij}, \quad i \in \mathcal{N}; \\
& p_i \in \mathcal{P}_i, \quad i \in \mathcal{N}^+; \\
& V_0 = V_0^{\text{ref}}; \\
& \underline{V}_i \leq V_i \leq \overline{V}_i, \quad i \in \mathcal{N}^+.
\end{aligned}$$

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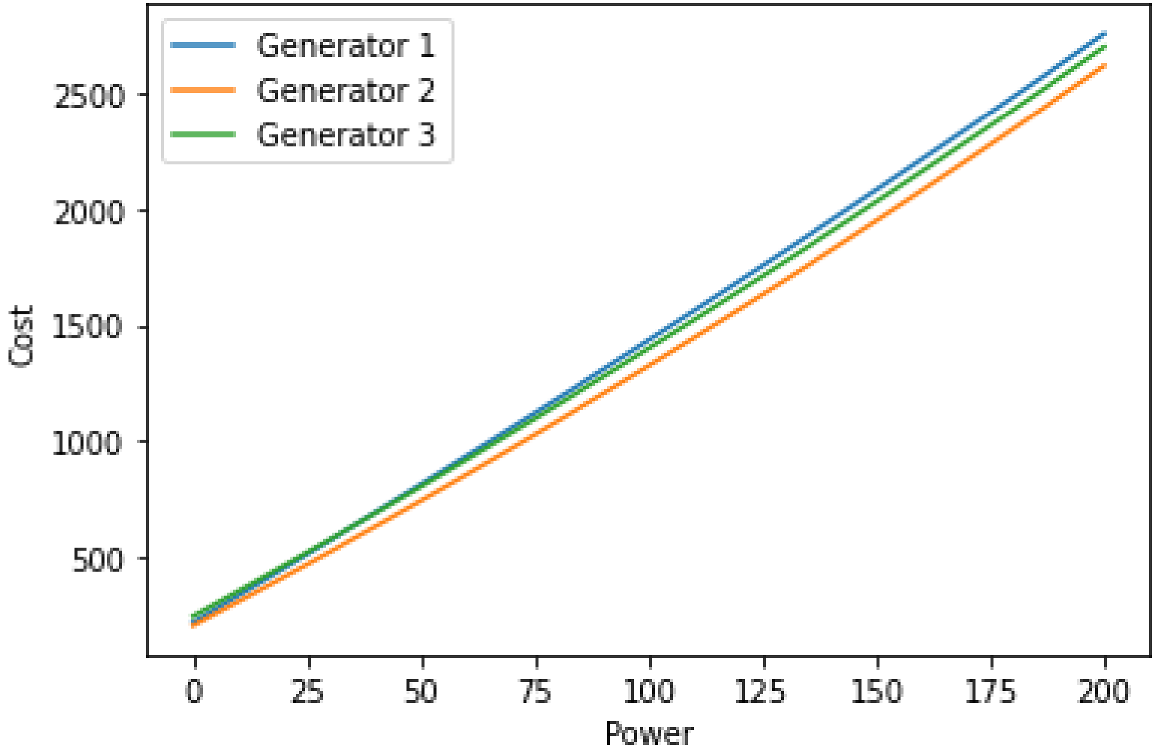
Cost functions



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Convex formulation

$$
\begin{aligned}
\mathbf{OPF}' : \ \min \ & \sum_{i \in \mathcal{N}} f_i(p_i) \\
\text{over } & p_i \in \mathbb{R}, v_i \in \mathbb{R} \text{ for } i \in \mathcal{N}; \\
& W_{ij} \in \mathbb{R}^+ \text{ for } i \sim j, \\
\text{s.t. } p_i = & \sum_{j : j \sim i} (v_i - W_{ij}) y_{ij}, \quad i \in \mathcal{N}; && \text{(6a)} \\
& p_i \in \mathcal{P}_i, \quad i \in \mathcal{N}^+; && \text{(6b)} \\
& v_0 = \left[V_0^{\text{ref}}\right]^2; && \text{(6c)} \\
& \underline{V}_i^2 \le v_i \le \overline{V}_i^2, \quad i \in \mathcal{N}^+; && \text{(6d)} \\
& W_{ij} = W_{ji}, \quad i \to j; && \text{(6e)} \\
& \begin{bmatrix} v_i & W_{ij} \\ W_{ji} & v_j \end{bmatrix} \succeq 0, \quad i \to j; && \text{(6f)} \\
& \operatorname{rank} \begin{bmatrix} v_i & W_{ij} \\ W_{ji} & v_j \end{bmatrix} = 1, \quad i \to j. && \text{(6g)}
\end{aligned}
$$

Exact SOCP Relaxation

- If an optimal SOCP solution (p, v, W)satisfies (6g), then (p, v, W) also solves OPF′.
- Furthermore, compute V as
  - $V_i = \sqrt{v_i}$
- then it can be shown that (p, v)solves OPF