Measures of cost for (electrical) energy

The Levelized Cost of Electricity (LCOE) - or Levelized Energy Cost (LEC) is often taken as a measure for defining the cost of electrical energy. It is the net present value of the unit-cost of electricity.

LCOE is often taken as a proxy for the average price that the generating asset must receive in a market to break even over its lifetime. It is a first-order economic assessment of the cost competitiveness of an electricity-generating system that incorporates all costs over its lifetime: initial investment, operations and maintenance, cost of fuel, cost of capital.

$$LCOE = \frac{cost}{electricity} = \frac{\sum_{t=1}^{n} \frac{I_t + M_t + F_t}{(1+r)^t}}{\sum_{t=1}^{n} \frac{E_t}{(1+r)^t}}$$

where:

I_t = Investment expenditures in the year t

M_t = Operations and maintenance expenditures in the year t

F_t = Fuel expenditures in the year t

E_t = Electricity generation in the year t

r = Discount rate

n = Life of the system

The net present value (NPV) of a project for electricity generation is defined as:

$$NPV = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

where C_n is the cash flow during year n. C_n is equal to $R_t - M_t - F_t - I_t$ where R_t are the revenues generated by the power plant during year t.

The internal rate of return (IRR) of a project is the value of r that leads to a NPV equal to 0:
$NPV(r) = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} = 0$

The payback period is the period of time required to recoup the funds expended in an investment.

Exercice: Mister X has installed at home 4 kWp of PV panels at a price of 6000 €. His panels have a lifetime of 20 years. This installation generates 3500 kWh of electricity per year.
[A] Compute the LCOE given a discount rate of 0% and 5%.
[B] Assume a retail price for electriciy of 23 c/kWh, compute the payback period of the installation.
[C] Given the same retail price for electricity, compute the internal rate of return of the project.
Reminder: $\sum_{k=a}^{b} q^k = \frac{q^a - q^{b+1}}{1-q}$ where $a, b \in \mathbb{N}$ and $q \neq 1$.

[A] We have: (i) $I_1 = 4000$ € and $I_t = 0$ if $t \neq 1$ (ii) $M_t = 0$, $F_t = 0$, $E_t = 3500\ \forall t$ (iii) $n = 20$.
If $r = 0$, we have LCOE$=\frac{6000}{3500\times 20}=$ 8.5 c/kWh.
If $r \neq 0$, the LCOE can be rewritten as:

$$
\begin{aligned}
LCOE &= \frac{\frac{6000}{(1+r)}}{\sum_{t=1}^{20} \frac{3500}{(1+r)^t}} = \frac{6000 \times q}{3500 \times \sum_{t=1}^{20} q^t} \\
&= \frac{6000 \times q}{3500 \frac{q-q^{21}}{1-q}}
\end{aligned}
$$

where $q = \frac{1}{1+r}$. If $r = 0.05$, we have $q = 0.952$ and LCEO $= \frac{5712}{3500\times 12.42} =$ 13.1 c/kWh. If $r = 0.10$, we have $q = 0.909$ and LCEO $= \frac{5454}{3500\times 8.50}$ 18.3 c/kWh.

[B] Every year, the installation is generating $0.23 \times 3500 = 805$ € worth of electricity. The installation costs 6000 €. The payback time is therefore equal to $\frac{6000}{805}$ =7.45 years.

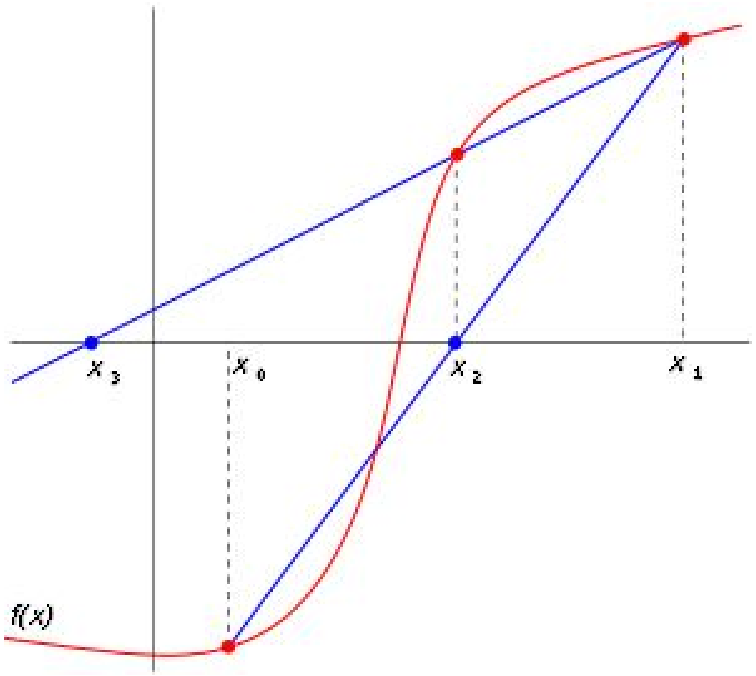
[C] No closed-loop solution. How to proceed?

A side note on the computation of the IRR

Computing the IRR is equivalent to finding the value of r that satisfies the equation $NPV(r) = 0$. In the general case, no closed form solution exists. A finite difference approximation of the Newton-Raphson method can however be used for finding a solution to this equation:

$$r_{n+1} = r_n - \frac{NPV(r_n)}{\frac{NPV(r_n) - NPV(r_{n-1})}{r_n - r_{n-1}}}$$

where r_n is considered the n^{th} approximation of the IRR.



The first two iterations of the Newton's method for finding the root of the function $f(x)$

The convergence behaviour of the sequence is the following:

- If the function $NPV(r)$ has a single real root IRR, then the sequence converges reproducibly towards IRR.

- If the function $NPV(r)$ has n real roots IRR_1, IRR_2, $\ldots$, IRR_n, then the sequence converges to one of the roots, and changing the values of the initial pairs may change the root to which it converges.

- If function NPV(r) has no real roots, then the sequence tends towards $+\infty$.

Exercice: Write a small program for computing the IRR of previous exercise and illustrate the results obtained.

```python
#Program for computing the IRR of the PV installation


def NPVExample(r):
    C = [None]*20
    C[0]=3500*0.23-6000
    for n in range(1,20):
        C[n]=3500*0.23
    NPV=0
    n=0
    while n < len(C):
        NPV=NPV+C[n]/(1+r)**(n+1)
        n=n+1
    return NPV


def NewtonRaphston(x0, x1, f, iterMax, accuracy):
    iter=0
    xnMinus1=x0
    xn=x1
    while (f(x0) != f(x1)) & (iter < iterMax) &  (f(x1) < accuracy):
        xnPlus1=xn-f(xnMinus1)/((f(xn)-f(xnMinus1)/(xn-xnMinus1)))
        iter=iter+1
    return xnPlus1



print 'The value of the IRR for the project is '
print  NewtonRaphston(x0=0, x1=10, f=NPVExample,iterMax=1000, accuracy=0.00001)
```

If you run this program in Python, you will get an IRR of 16.84%

The levelized cost of electricity for some newly built renewable and fossil-fuel based power stations in euro per kWh in Germany (estimation done in 2013 by the Fraunhofer Institute):