Energy Markets

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Exercise Session 2: Day-Ahead electricity markets - Advanced

The aim of this exercise session is to appraise and better understand the basic structure of electricity markets, and most particularly its day-ahead mechanism. It is a direct continuation of the Exercise session 1.

Problem 1: Problem 1: Formulating the market clearing more mathematically

Consider the market setup and list of supply offers of Problem 2 (in Exercise session 1), while assuming that the electric power demand to be met is fixed to 180MWh.

1.1 What is the most simple way to find the equilibrium point? Intuitively, what is the clearing price, who will produce and how much?
Draw a vertical straight line at the desired quantity of energy. The clearing price is 15 €/MWh and producers G_3, G_5 and G_2 will be producing 32, 70 and 78 MWh respectively (Figure 1).

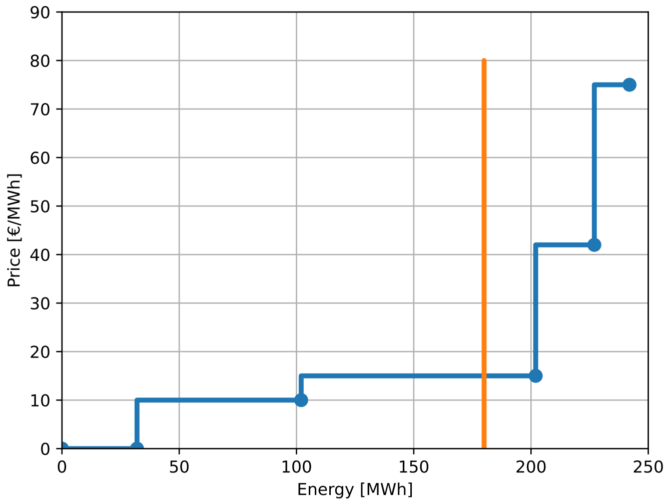

Figure 1: Supply curve with inelastic demand

1.2 Since demand is fixed, what is the objective of the market clearing with the supply side? Write it as an objective function. Is it a maximization or minimization problem?
As we assume an inelastic demand, the market clearing problem that aims at maximizing social welfare is equivalent to minimizing total production cost. The objective function is:

$$\min_{y_j^G} \sum_j \lambda_j^G y_j^G$$

1.3 What is the balance condition for the market (between supply and demand)? Write it as a balance constraint.
The balance condition is that supply should constantly meet demand. The constraint is:

$$\sum_j y_j^G = D$$

1.4 Deduce the complete linear program to be used for clearing the market.

$$\min_{y_j^G} \sum_j \lambda_j^G y_j^G$$
$$\text{s.t.} \sum_j y_j^G = D$$
$$0 \leq y_j^G \leq P_j^{max}, \forall j \in J$$

As an extension, we now consider that the list of demand offers that is given in Problem 3 of Exercise session 1.

1.5 What should be the objective function of the market-clearing (since having to consider both supply and demand sides)? Write it as an objective function. Is it a maximization or minimization problem?
Taking the demand offers given in Problem 3 of Exercise session 1, the demand curve is not a straight line anymore. Therefore the configuration of the market clearing is the following:

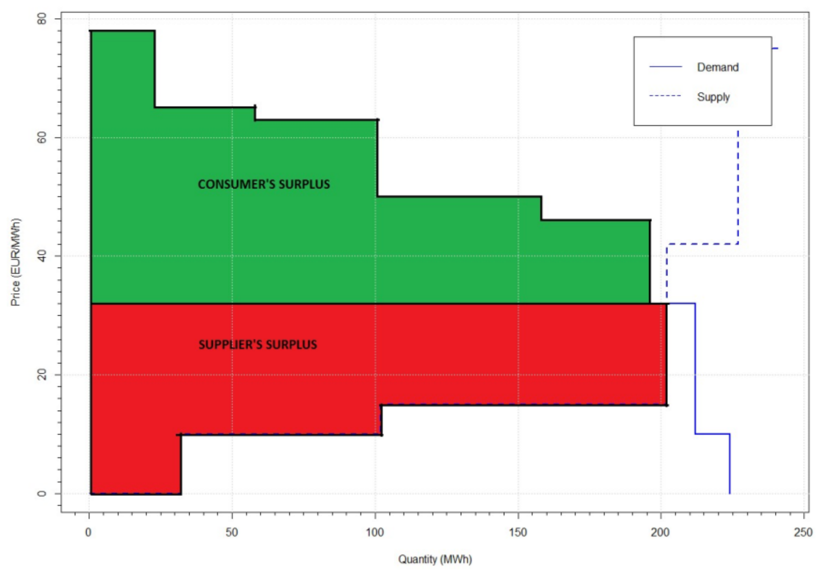

Figure 2: Social welfare maximization with demand/supply offers

As we consider both supply and demand sides, the problem is transformed into a maximization problem (Figure 2). The demand is not a fixed parameter but a set of demand offers that are variables in the optimization problem (notion of "willingness to pay"). The objective function becomes:

$$\max_{y_i^D, y_j^G} \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G.$$

1.6 What is the balance condition for the market (between supply and demand)? Write it as a balance constraint.
The balance condition is that supply should constantly meet demand. The constraint is

$$\sum_j y_j^G = \sum_i y_i^D.$$

1.7 Deduce the complete linear program to be used for clearing the market.

$$\max_{y_i^D, y_j^G} \sum_i \lambda_i^D y_i^D - \sum_j \lambda_j^G y_j^G$$
$$\text{s.t.} \sum_j y_j^G = \sum_i y_i^D$$
$$0 \leq y_j^G \leq P_j^G, \forall j \in J$$
$$0 \leq y_i^D \leq P_i^D, \forall i \in I$$

Feel free to implement those linear programs in Python/R/Matlab/GAMS/etc. in order to verify that you obtain the same solution as in Problem 4 of Exercise session 1.

Problem 2: Settlement and revenues

For this problem, you should consider the list of supply and demand offers presented in session 1.

2.1 Look through the lecture slides, and define the difference between "pay-as-bid" and "uniform pricing".
Pay as bid means that the producer will get each unit of energy sold, paid at the price he bid in the market. Uniform pricing is the one used for Nord pool. The market is cleared with a common price at which the supply and demand curves are crossing and everyone is selling for that price. Even if their bids were lower than the market price.

2.2 Determine the revenues of various market participants on the supply side under uniform pricing settlement. What if using pay-as-bid instead?

Looking at the merit order computed in Problem 4 of exercise session 1, the 3 suppliers to be scheduled are G_3, G_5 and G_2. The quantity sold and the market price are also determined from this merit order.

Supplier name	Supplier id.	Quantity sold [MWh]	Bidding price [€/MWh]	Market Price	Revenue under "Uniform pricing" [€/MWh] [€]	Revenue under Pay-as-Bid" €
ShinyPower	G_3	32	0	32	1024	0
BlueWater	G_5	70	10	32	2240	700
Nuke22	G_2	100	15	32	3200	1500

Table 1: Supply side revenues

2.3 Determine the payments for various market participants on the demand side under uniform pricing settlement. What if using pay-as-bid instead? Looking at the merit order computed in Problem 4 of exercise session 1, the only demand that is not scheduled at all is D_3. The quantity purchased and the market price are also determined from this merit order.

Problem 3: Day-ahead market with 2 zones

Let us now complexify a bit the market set-up and make it more realistic. Our market is now split into two zones (West and East). The various suppliers and demands are associated to these zones as follows:
The available transmission capacity between these 2 zones is of 30MW. In the following we will assess how this may affect the previous market clearing and revenues that were obtained when not having such transmission constraints.

Demand name	Demand id.	Quantity purchased [MWh]	Bidding price [€/MWh]	Market Price	Payment under "Uniform pricing" [€/MWh] [€]	Payment under Pay-as-Bid" €
CleanCharge	D_2	23	78	32	736	1794
WeLovePower	D_1	35	65	32	1120	2275
QualiWatt	D_5	43	63	32	1376	2709
El-Forbundet	D_7	57	50	32	1824	2850
ElRetail	D_4	3	46	32	1216	1748
IntelliWatt	D_6	6	32	32	192	192

Table 2: Demand side payments

Supplier Name	Supplier id.	Zone	Quantity [MWh]	Price [€/MWh]
Flexigas	G_1	East	15	75
Nuke22	G_2	West	100	15
ShinyPower	G_3	East	32	0
RoskildeCHP	G_4	East	25	42
BlueWater	G_5	West	70	10

Demand Name	Demand id.	Zone	Quantity [MWh]	Price [€/MWh]
WeLovePower	D_1	East	35	65
CleanChange	D_2	East	23	78
JyskeEl	D_3	East	12	10
ElRetail	D_4	East	38	46
QualiWatt	D_5	West	43	63
IntelliWatt	D_6	East	16	32
El-Forbundet	D_7	West	57	50

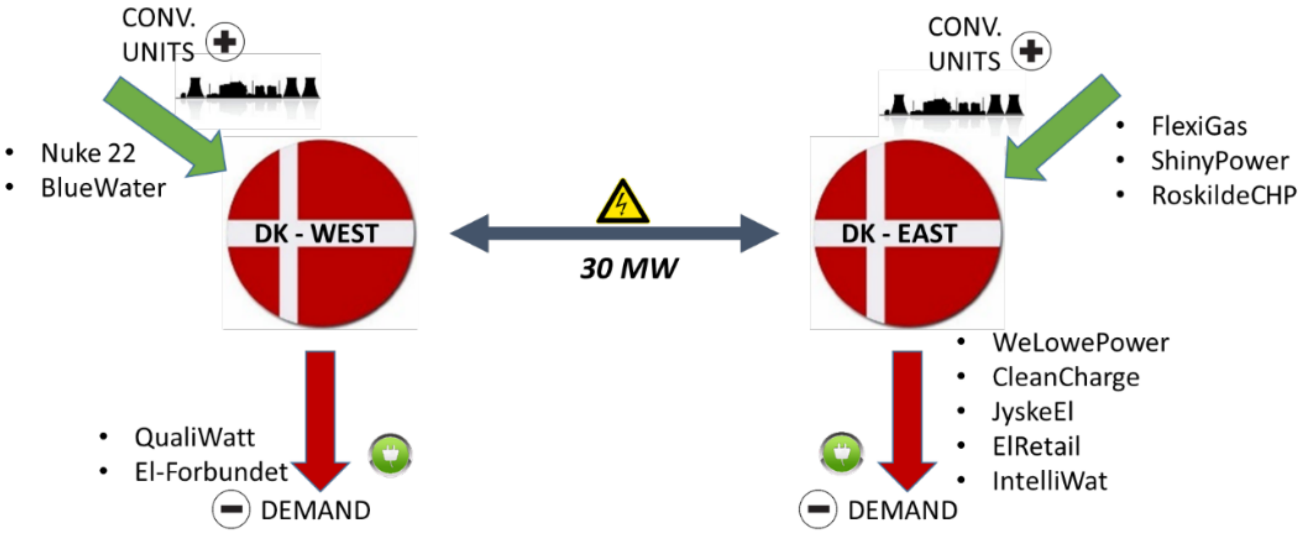

Figure 3: Schematic representation of the system layout

3.1 Make a schematic representation of the system layout (i.e., the two zones with its players, as well as the transmission constraints between these two).

3.2 Assess whether the previous market clearing (from Problem 1) is feasible or not by comparing the scheduled supply and demand locally and the amount of energy that should be transmitted between the two zones.

Based on previous clearing and resulting schedules for the various suppliers and demands, one can map them on both West and East side:

- Supply on the West side: G_5 with 70MWh and G_2 with 100MWh.
- Demand on the West side: D_5 with 43MWh and D_7 with 57MWh.
- Supply on the East side: G_3 with 32MWh.
- Demand on the East side: D_2 with 23MWh, D_1 with 35MWh, D_4 with 38MWh and D_6 with 6MWh.

This translates to a total supply of 170MWh on the West side, for a total demand of 100MWh. On the East side, the total supply is of 32MWh, and the demand is of 102MWh. Consequently, there would be a need to transmit 70MWh from the West to the East. This is definitely not feasible in view of the capacity of 30MW of the West-East transmission link.

3.3 Obtain the supply and demand curves for both zones (as if there was no connection between the two).

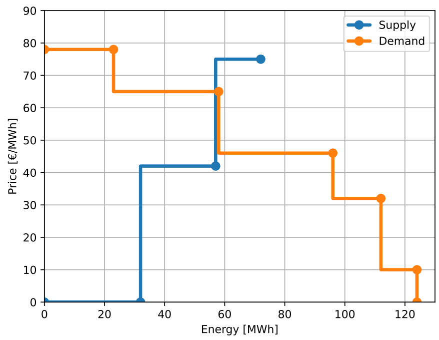

Figure 4: Demand/supply curve for DK-EAST

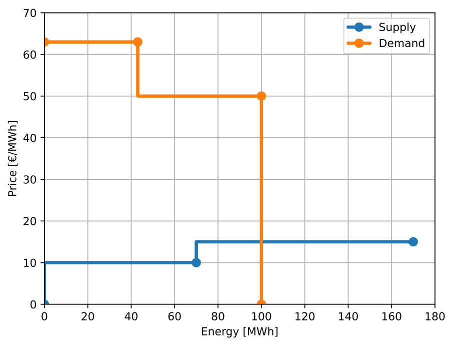

Figure 5: Demand/supply curve for DK-WEST

3.4 Update this curves by adding extra virtual offers representing transfer of power from one zone to the next. From which and to which zone should the power flow?
From Figure 4 and Figure 5 we can notice that DK-EAST is the high-price area and DK-WEST the low-price area. It is know that if transmission between areas is possible, energy should flow from the low-price area to the high-price area (DK-WEST $\rightarrow_{\text{Power flow}}$ DK-EAST). Therefore the transmission is Power flow added on the supply curve for the high-price area (Figure 6) and to the demand curve on the low-price area (Figure 7).

3.5 Determine equilibrium price in both zones as well as revenues and payments for each market participant when

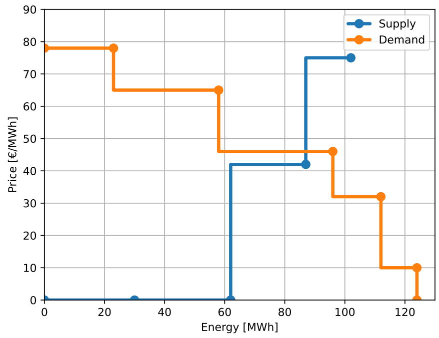

Figure 6: Demand/supply curve for DK-EAST with transmission line

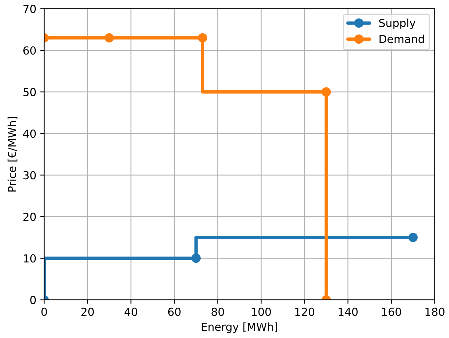

Figure 7: Demand/supply curve for DK-WEST with transmission line

(a) There is no transmission between the two zones.
Equilibrium price in DK-EAST (without transmission): 65€/MWh (Figure 4)
Equilibrium price in DK-WEST (without transmission): 15€/MWh (Figure 5)

Supplier Name	Supplier id.	Zone	Quantity sold [MWh]	Market price [€/MWh]	Revenue [€]
Flexigas	G_1	East	0	65	0
ShinyPower	G_3	East	32	65	2080
RoskildeCHP	G_4	East	25	65	1625
Nuke22	G_2	West	30	15	450
BlueWater	G_5	West	70	15	1050

Table 3: Producer's revenues without transmission

Demand Name	Demand id.	Zone	Quantity purchase [MWh]	Market price [€/MWh]	Payment [€]
WeLovePower	D_1	East	34	65	2210
CleanChange	D_2	East	23	65	1495
JyskeEl	D_3	East	0	65	0
ElRetail	D_4	East	0	65	0
IntelliWatt	D_6	East	0	65	0
QualiWatt	D_5	West	43	15	645
El-Forbundet	D_7	West	57	15	855

Table 4: Consumer's payments without transmission

(b) There is 30 MW of transmission between the two zones.
Equilibrium price in DK-EAST (with transmission): 46€/MWh (Figure 6)
Equilibrium price in DK-WEST (with transmission): 15€/MWh (Figure 7)

Supplier Name	Supplier id.	Zone	Quantity sold [MWh]	Market price [€/MWh]	Revenue [€]
Flexigas	G_1	East	0	46	0
ShinyPower	G_3	East	32	46	1472
RoskildeCHP	G_4	East	25	46	1150
Nuke22	G_2	West	60	15	900
BlueWater	G_5	West	70	15	1050

Table 5: Producer's revenues with transmission

Demand Name	Demand id.	Zone	Quantity purchase [MWh]	Market price [€/MWh]	Payment [€]
WeLovePower	D_1	East	35	46	1610
CleanChange	D_2	East	23	46	1058
JyskeEl	D_3	East	0	46	0
ElRetail	D_4	East	29	46	1334
IntelliWatt	D_6	East	0	46	0
QualiWatt	D_5	West	43	15	645
El-Forbundet	D_7	West	57	15	855

Table 6: Consumer's payments with transmission

3.6 Compare the prices with the case where there was no transmission constraint at all (i.e. as if the two zones where one).
With no transmission the price in DK-WEST remains unchanged. However, in DK-EAST the price is increased to 65 €/MWh

3.7 What would be the minimum transmission capacity needed here for the price to be the same in the 2 zones?
In this case, the minimum capacity should be 70 MW to have the same price in both zones. This is since it represents the power deficit on the East side and power surplus on the West side. With a 70 MW transmission, one would not need to split the market and would then end up with a single price for all.

Problem 4: Extract and analyse data for a day-ahead market

Besides some of the basic modelling and market concepts dealt with through the previous problems, a key aspect of working with electricity markets (including the day-ahead stage) is to develop an ability to find and analyse relevant data. In the present problem, emphasis is then placed on extracting data from the Nord Pool website in order to appraise what is going on there.

4.1 Pay a visit to the market data page of the Nord Pool website and have a look at prices in tables in chart for the last cleared day. How similar are prices for the 2 market areas of Denmark? What are the daily variations, and can you explain them?

4.2 One may also download more extensive datasets from the historical market data webpage of the Nord Pool website. There you may for instance get some of the data for 2020 so far:

- hourly consumption data used at the time of clearing the market,
- hourly wind power forecasts used at the time of clearing the market,
- hourly market prices as the result of the market-clearing process.

Download these data and choose your favorite data analysis environment (Python/R/Matlab/Excel/etc,).

4.3 Find a typical day with high wind power production in DK1, and look at the corresponding prices. Do the same with a typical day with very low wind power production. Is there something to learn here?

4.4 What is the average day-ahead, also called spot prices, for DK1 (Western Denmark) as a function of the time of the day? Its maximum and minimum? Are they defined limits for these minimum and maximum values (i.e., as set by the market rules)?

4.5 What is the average consumption for DK1 and DK2 (Eastern Denmark) as a function of the time of the day?

Corrections are omitted for that part. Results for that exercise may be subject to feedback on an individual or group basis instead.