



ELEC0018-1

Energy Markets

Lecture 5 : Day-ahead market
with transmission

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Previous week

- Day-ahead market **principles** and the EPEX SPOT clearing and settlement mechanism.
- Formulation of the optimal day-ahead **dispatch** problem (primal form).
- Formulation of the day ahead **settlement** problem (dual form).
- Particular case of **inelastic demand**.
- Influence of **RES** on the clearing prices.

Optimisation problem - Reminder

Demand bids:

- N_D demand bids: set of offers $L_D = \{D_i | i = 1, \dots, N_D\}$
- Quantity for offer D_i : E_i^D
- Price for offer D_i : p_i^D

Generation bids:

- N_G demand bids: set of offers $L_D = \{G_j | j = 1, \dots, N_G\}$
- Quantity for offer G_j : E_j^G
- Price for offer G_j : p_j^G

Variables of the problem:

- Consumption level y_i^D : how much energy is cleared from bid D_i at price p_i^D
- Generation level y_j^G : how much energy is cleared from bid G_j at price p_j^G

Optimisation problem - Reminder

$$\max_{\{y_i^D\}, \{y_j^G\}} \sum_{i=1}^{N_D} p_i^D y_i^D - \sum_{j=1}^{N_G} p_j^G y_j^G$$

subject to

$$\sum_{i=1}^{N_D} y_i^D - \sum_{j=1}^{N_G} y_j^G = 0$$
$$0 \leq y_i^D \leq E_i^D, \quad \forall i = 1, \dots, N_D$$
$$0 \leq y_j^G \leq E_j^G, \quad \forall j = 1, \dots, N_G$$

The solution to the problem is the **optimal dispatch** for the market participants: the sequence of generation and consumption levels $\{y_i^{D*}\}, \{y_j^{G*}\}$ of each offer maximising social welfare.

Optimisation problem - Reminder

$$\min_{v_{eq}, \{v_i^D\}, \{v_j^G\}} \sum_{i=1}^{N_D} v_i^D E_i^D - \sum_{j=1}^{N_G} v_j^G E_j^G$$

subject to

$$\begin{aligned} v_i^D + v_{eq} &\geq p_i^D, & \forall i = 1, \dots, N_D \\ v_j^G - v_{eq} &\geq -p_j^G, & \forall j = 1, \dots, N_G \\ v_i^D &\geq 0, & \forall i = 1, \dots, N_D \\ v_j^G &\geq 0, & \forall j = 1, \dots, N_G \end{aligned}$$

The optimal value of the dual variable v_{eq}^* gives the market price.

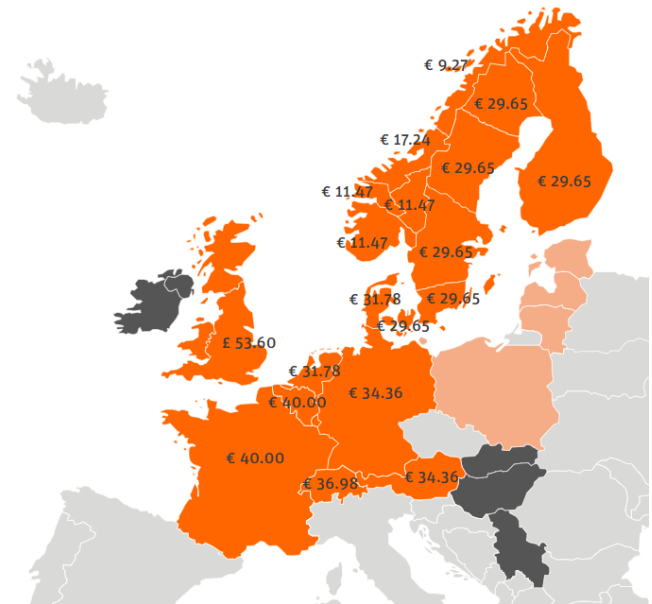
Transmission networks

Many European countries can bid on EPEX SPOT.

What would happen if all the energy is generated in Belgium and consumed in the Netherlands? Is it possible to transmit the energy over the network?

The **network** is divided into **nodes** sharing the same day-ahead price. The nodes are interconnected with **transmission lines**. It is possible to transmit a limited amount of power from one node to the other through these lines.

How to take this constraint into account in the day-ahead auction?

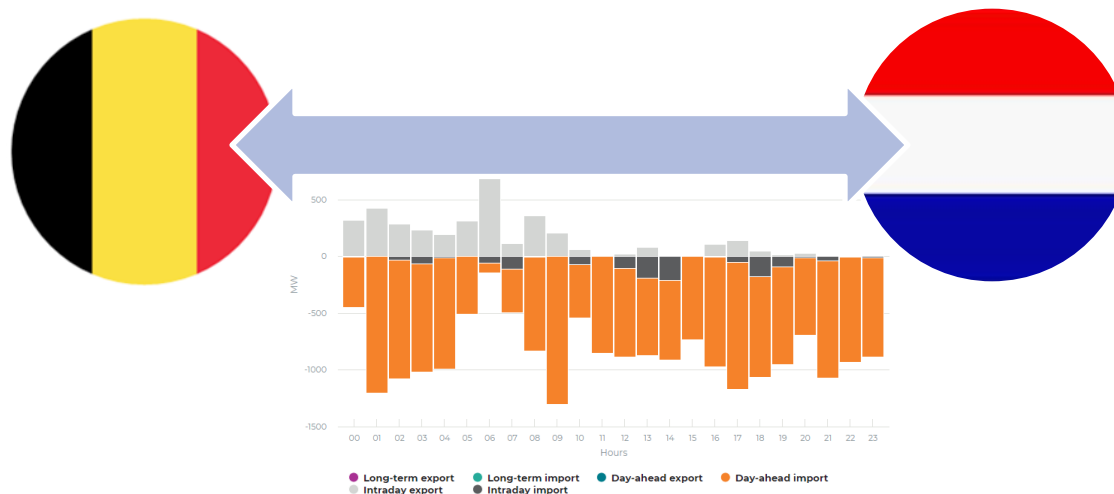


https://www.epexspot.com/en/market-data?market_area=&trading_date=2020-09-06&delivery_date=2020-09-07&underlying_year=&modality=Auction&sub_modality=DayAhead&product=60&data_mode=map&period=

Transmission networks - Solution

The objective remains to maximize the total social welfare over the different nodes.

The energy exchanged between two nodes is limited by the **maximal capacity** of the transmission line connecting them.



Optimisation problem - Formulation

Each offer is now associated with a node $n = 1, \dots, N$:

Demand bids

- N_D demand bids: set of offers $L_D = \{D_i^n | i = 1, \dots, N_D; n = 1, \dots, N\}$
- Set of indices of demand bids in node n : $I_n = \{i | D_i^{n'} \in L_D; n' = n\}$
- Quantity for offer D_i^n : E_i^D
- Price for offer D_i^n : p_i^D
- Node for offer D_i^n : n

Generation bids

- N_G generation bids: set of offers $L_G = \{G_j^n | j = 1, \dots, N_G; n = 1, \dots, N\}$
- Set of indices of generation bids in node n : $J_n = \{j | G_j^{n'} \in L_G; n' = n\}$
- Quantity for offer G_j^n : E_j^G
- Price for offer G_j^n : p_j^G
- Node for offer G_j^n : n

Optimisation problem - Formulation

Maximal capacities $C_{n,n'}$ of the interconnections between two nodes n and n' are taken into account.

Variables of the problem:

- Consumption level y_i^D of offer D_i^n : how much energy is cleared from the bid D_i^n at price p_i^D
- Generation level y_j^G of offer G_j^n : how much energy is cleared from the bid G_j^n at price p_j^G
- Power exchanged $q_{n,n'}$ between the nodes n and n' (> 0 when power flows from n to n').

Optimisation problem

The objective is still to maximise social welfare:

$$\max_{\{y_i^D\}, \{y_j^G\}} \sum_{i=1}^{N_D} p_i^D y_i^D - \sum_{j=1}^{N_G} p_j^G y_j^G$$

Each bid can still be partially accepted but is bounded by the quantity offered at that price:

$$\begin{aligned} 0 \leq y_i^D \leq E_i^D, & \quad \forall i = 1, \dots, N_D \\ 0 \leq y_j^G \leq E_j^G, & \quad \forall j = 1, \dots, N_G \end{aligned}$$

Optimisation problem

The **exchanged power** between two nodes is **bounded** by the maximal capacity of the line. The power can nevertheless flow in both directions:

$$-C_{n,n'} \leq q_{n,n'} \leq C_{n,n'}, \quad \forall n, n' = 1, \dots, N$$

The power flowing from node n to node n' is the opposite of the power flowing from node n' to node n :

$$q_{n,n'} = -q_{n',n}, \quad \forall n, n' = 1, \dots, N$$

At each node, the difference between consumption and production equals the total power exchanged by this node with the other nodes:

$$\sum_{i \in I_n} y_i^D - \sum_{j \in J_n} y_j^G = \sum_{n'=1}^N q_{n',n}, \quad \forall n = 1, \dots, N$$

Optimisation problem

The solution to the problem is the **optimal dispatch** subject to the transmission constraints.

$$\max_{\{y_i^D\}, \{y_j^G\}} \sum_{i=1}^{N_D} p_i^D y_i^D - \sum_{j=1}^{N_G} p_j^G y_j^G$$

subject to

$$\begin{aligned} \sum_{i \in I_n} y_i^D - \sum_{j \in J_n} y_j^G &= \sum_{n'=1}^N q_{n',n}, & \forall n = 1, \dots, N \\ 0 \leq y_i^D &\leq E_i^D, & \forall i = 1, \dots, N_D \\ 0 \leq y_j^G &\leq E_j^G, & \forall j = 1, \dots, N_G \\ -C_{n,n'} \leq q_{n,n'} &\leq C_{n,n'}, & \forall n, n' = 1, \dots, N \\ q_{n,n'} &= -q_{n',n}, & \forall n, n' = 1, \dots, N \end{aligned}$$

Optimal dispatch - Discussion

The power $q_{n,n'}$ transmitted on a line can be expressed as a function of the susceptance of the line, $B_{n,n'}$, and of the difference of phase between the voltages at the nodes, $\delta_{n,n'}$:

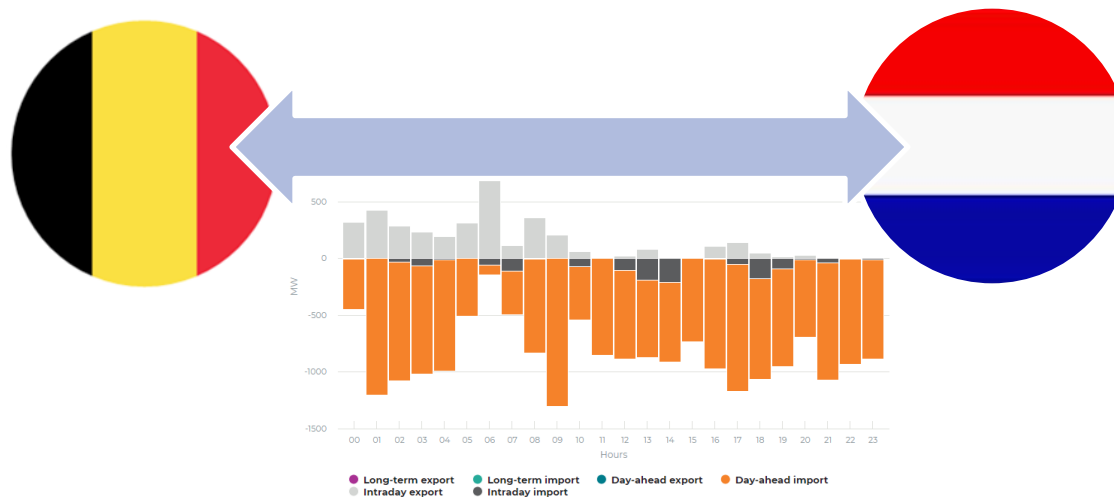
$$q_{n,n'} = B_{n,n'} \delta_{n,n'}$$

In a two-node system, the power flows from the cheapest node (if cleared independently) towards the most expensive node. It is not true when there are more than two nodes.

Exercise

Simplify the previous model to a two-node system.

- Consider a transmission line with capacity C .
- Let q be the power exchanged between the two lines.



Exercise - Solution

$$\max_{\{y_i^D\}, \{y_j^G\}} \sum_{i=1}^{N_D} p_i^D y_i^D - \sum_{j=1}^{N_G} p_j^G y_j^G$$

subject to

$$\begin{aligned} \sum_{i \in I_1} y_i^D - \sum_{j \in J_1} y_j^G &= q \\ \sum_{i \in I_2} y_i^D - \sum_{j \in J_2} y_j^G &= -q \\ 0 \leq y_i^D \leq E_i^D, \quad \forall i = 1, \dots, N_D \\ 0 \leq y_j^G \leq E_j^G, \quad \forall j = 1, \dots, N_G \\ -C \leq q \leq C \end{aligned}$$

Merit order

- Exercise

Build the merit orders for both nodes assuming first they are not interconnected.

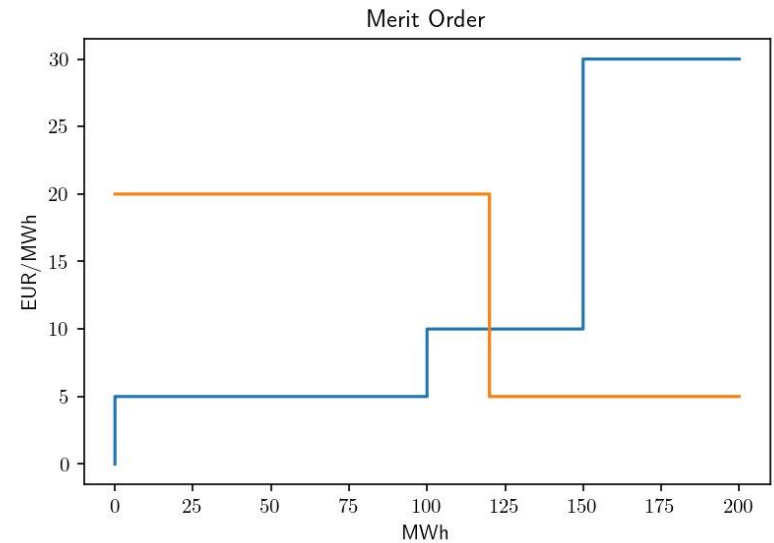
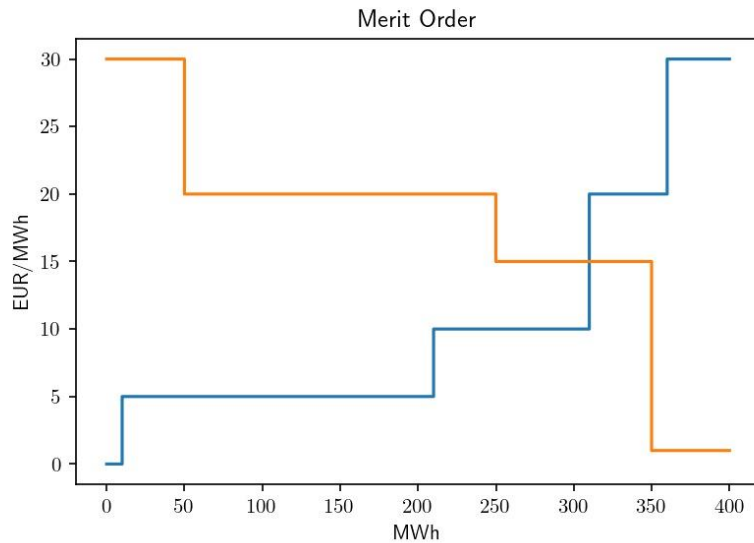
Assuming we can transfer from node 1 to node 2 10 MWh, is it profitable to trade from one node to the other given their independent merit orders?

Sell/Buy	Node	Capacity for hour h	Price of the energy
Sell	1	50 MWh	20 €/MWh
	1	100 MWh	10 €/MWh
	1	20 MWh	30 €/MWh
	1	200 MWh	5 €/MWh
	1	10 MWh	0 €/MWh
	2	100 MWh	30 €/MWh
	2	50 MWh	10 €/MWh
	2	100 MWh	5 €/MWh
Buy	1	50 MWh	1 €/MWh
	1	100 MWh	15 €/MWh
	1	200 MWh	20 €/MWh
	1	50 MWh	30 €/MWh
	2	100 MWh	5 €/MWh
	2	120 MWh	20 €/MWh

Merit Order - Solution



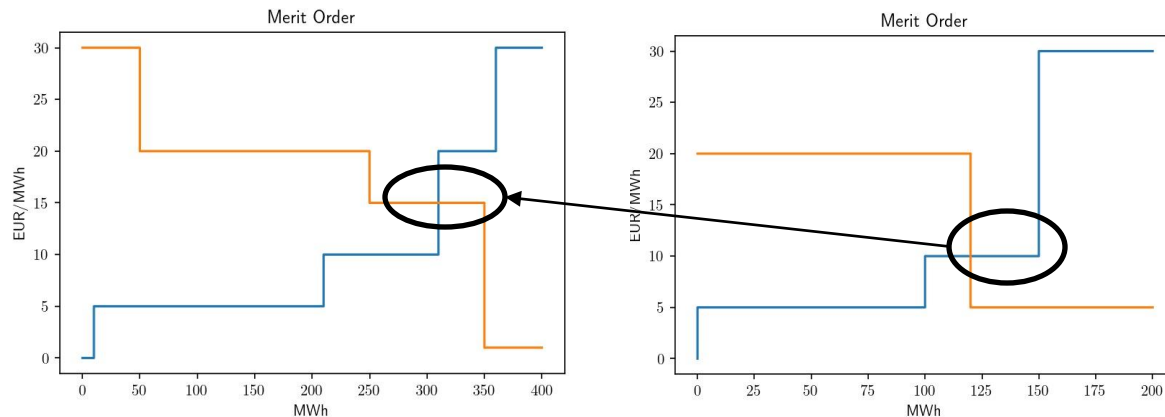
Build the merit orders for both nodes assuming first they are not interconnected.



Merit Order - Solution

Assuming we can transfer from node 1 to node 2 10 MWh, is it profitable to trade from one node to the other given their independent merit orders?

The **30 MWh** available at the second node are **less expensive** than the system price at the first node. It will thus be profitable to additionally transfer 10 MWh from node two to node one. In this example, the equilibrium prices remain unchanged. The merit order at node one is **shifted** by 10 MWh !



Settlement

Nodal prices : equilibrium prices of the merit order at each nodes. The nodal prices are different if the transmission lines are saturated.

In Europe, the zones sharing the same prices do not exactly correspond to the nodes of the network, meaning we do not consider all sources of congestion in the network.

Zonal prices: prices in such a clearing zone.

In Europe, each participant pays or is paid at its zonal price.

Settlement - Congestion surplus

Congestion surplus : difference between the payments made by the loads and the revenues of the generator.

Congestion surplus only arises when the transmission line is saturated/congested.

Settlement - Computation

The zonal prices correspond to the **optimal dual variables** v_n^* paired with the equilibrium constraints between production and consumption in each zone.

These optimal dual variables represent the **marginal cost** of buying the last unit of power in a zone.

Limitations

- When a bid is partially cleared, is it feasible to deliver the fraction cleared? For example, is it possible to use a big coal engine to produce a little of energy?
- Is it possible to change the production from one hour to another instantaneously? Do you think a coal engine can be started very fast?
- Are the costs independent on the state of the production plant?
- There are many **operational constraints** !

More advanced EPEX products - Overview

Classic **block orders** : several hours at the same price.

Big blocks : larger than classic blocks with the maximum size going up to 1300 MW.

Curtable blocks: set of blocks which can be either entirely executed or entirely rejected.

Linked blocks: set of blocks with a linked execution constraint, meaning the execution of one block depends on the execution of its father block. They allow to represent the variation of electricity generation with regards to the market price.

Exclusive blocks: group of blocks within which a maximum of one block can be executed, so you're your electricity is traded at the most profitable moment.

More at : <https://www.epexspot.com/en/tradingproducts>

More advanced EPEX products - Optimisation

- It is not straightforward to extend the market-clearing with block order.
- All 24-hourly products shall be **considered together**.
- It can be formulated as a Mixed Integer Linear Program (MILP).
- The **EUPHEMIA** algorithm is used to clear the EPEX market, more details at :
<https://www.n-side.com/wp-content/uploads/2017/08/Euphemia-Public-Presentation.pdf>

References

- <https://www.epexspot.com/>
- <https://www.creg.be/fr/professionnels/fonctionnement-et-monitoring-du-marche/plates-formes-du-marche/bourse-belge>
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- <https://www.epexspot.com/en/tradingproducts>
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